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M.P. Mukhina, I.V. Barkulova
(National Aviation University, Ukraine)

Model of one-dimensional search correlation extreme navigation system by relief field

Features of correlation extreme navigation are investigated. The model of one-dimensional search system is developed on the example of system working by relief field.

Introduction. For small unmanned aerial vehicles (UAV), the common practice is to use on board an information management complex with two basic navigation systems [1]: inertial navigation system (or strapdown one - SINS) and satellite navigation system (SNS).

The alternative variant to SNS working and giving the correction for INS is correlation extreme navigation system (CENS). The feature of correlation-extreme navigation on geophysical fields is the presence of certain anomalies or characteristic features of the field, which are random functions of time and space. Navigation is carried out by comparing the current implementation of the field with a reference field for which a known map is known. The main criterion for comparison is the correlation function, the extremum of which (maximum value) coincides with the most probable location of the object on the map.

Correlation extreme navigation system has a number of features related to the construction and functioning principle, in particular the dependence of the geophysical field on the coordinates of the location is substantially nonlinear, most often given tabularly in the form of cartographic information having the nature of the implementation of a random function. Here the principal mathematical model of CENS working by relief field is developed and researched.

Problem statement. Let's consider a one-dimensional variant of the search CENS working on relief field $f(x)$, which is given in the infinite interval $(-\infty, +\infty)$. At the beginning of the field reference, we take the coordinate x_{SINS}, which is given by SINS, and is measured discretely with the interval Δx.

The known (reference) realizations of the field S_i consist of N discrete field values at the corresponding points of space (Fig. 1) and can therefore be represented as a N-dimensional vector

$$\mathbf{S}_i = \begin{bmatrix} S_{i1} \\ S_{i2} \\ \vdots \\ S_{iN} \end{bmatrix} = \begin{bmatrix} f\left(x_0 + i\Delta x\right) \\ f\left(x_0 + (i+1)\Delta x\right) \\ \vdots \\ f\left(x_0 + (i+N)\Delta x\right) \end{bmatrix}, \tag{1}$$

where $i = 0,...,2N+1$.

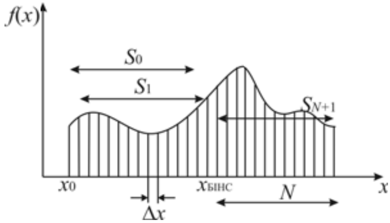


Fig. 1 Implementation of geophysical field at a given interval in the vicinity of the predicted SINS coordinate

The vector of CENS measurements **Z** can be represented as an additive mixture (in the first approximation) of the useful signal and noise δf and of the same dimension N:

$$\mathbf{Z} = \begin{bmatrix} f\left(x_0 + i\Delta x\right) + \delta\left(x_0 + i\Delta x\right) \\ f\left(x_0 + (i+1)\Delta x\right) + \delta\left(x_0 + (i+1)\Delta x\right) \\ \vdots \\ f\left(x_0 + (i+N)\Delta x\right) + \delta\left(x_0 + (i+N)\Delta x\right) \end{bmatrix} \tag{2}$$

The space of solutions (hypotheses) $\mathbf{G} = \{H_i\}$ coincides with the space of signals $\{\mathbf{S}_i\}$ [2]. The navigation solution is reduced to choosing the optimal hypothesis of D_{opt} based on the comparison of the input of measurement **Z** with all the signals (template realizations) $\mathbf{S}_i\left(i = 0,...,2N+1\right)$ and on the basis of available information about the probability characteristics of the noise δf and the a priori probabilities of the template realizations $\mathbf{S}_i$.

Let's assume that the law of error distribution of SINS is known, then the probability density function (pdf) of errors $p(\Delta)$ will determine the a priori probabilities of each of the template realizations $p(\mathbf{S}_i) = p_i$ as

$$p_i = \int\limits_{x_0 + i\Delta x}^{x_0 + (i+N)\Delta x} p(\Delta) d\Delta\ . \tag{3}$$

This assumption allows us to use the well-known Bayesian approach to checking hypotheses, in particular, let's denote the conditional pdf that the signal **Z** at the input matches to the template realization $\mathbf{S}_i$ as $p(H_i \mid \mathbf{Z})$. The risk of false decision that the signal at the input **Z** matches to the template realization $\mathbf{S}_j$, while in fact it matches to the template realization $\mathbf{S}_i$, is denoted as R_{ij}. The corresponding mathematical expectation of risk (loss) will be determined by the following expression:

$$\mathrm{E}\left[R_{ij}\right] = \sum_{i=1}^{2N+1} R_{ij}\, p\left(H_i \mid \mathbf{Z}\right). \tag{3}$$

The Bayesian approach minimizes the mathematical expectation of full losses on the set of solutions and is optimal [3].

Let's denote the pdf $p(\mathbf{Z} \mid H_i)$ of the signal $\mathbf{Z}$ at the input in the case when it matches to the template realization $\mathbf{S}_i$. Then the conditional probability will be determined by the well-known Bayesian formula as:

$$p(H_i \mid \mathbf{Z}) = \frac{p\left(H_i\right) p(\mathbf{Z} \mid H_i)}{p(\mathbf{Z})}. \tag{4}$$

However, it is possible to determine the unconditional pdf of the measured signal as $p(\mathbf{Z}) = \sum_{i=1}^{2N+1} p\left(H_i\right) p(\mathbf{Z} \mid H_i)$, then it can be entered into the formula for calculating the average losses $\mathrm{E}\left[R_{ij}\right] = \frac{1}{p(\mathbf{Z})} \sum_{i=1}^{2N+1} R_{ij}\, p(H_i)\, p\left(\mathbf{Z} \mid H_i\right)$ as a common factor and remove it from further consideration. Thus, the expression for average losses is as follows:

$$\mathrm{E}\left[R_{ij}\right] = \sum_{i=1}^{2N+1} R_{ij}\, p(H_i)\, p\left(\mathbf{Z} \mid H_i\right). \tag{5}$$

However, it is necessary to note the following feature of CENS, namely its multimodal distribution of the correlation function of the current field realization and template one, which results in the multimodal distribution of the probability that the signal $\mathbf{Z}$ at the input will be matched correctly to template $\mathbf{S}_i$.

In particular, let's consider the template realization of the relief field in the format SRTM30 [4], obtained by satellite radar topography. The data of the relief fields (Fig. 3) were presented in graphic form in the format *.GIF, where the corresponding gray hue in the range from 0 to 255 corresponds to the elevation of the relief in the range from -188 m to +5472 m above sea level with the mean height in 116.3 m and an average deviation of 253.9 m for a specific area of the ground surface (the region of the Eurasian continent with the territory of Ukraine, in particular the Carpathians with good informative mountain surface).

The data file contains geo-referencing information, including the following fields: 0.00833333333333 - the dimension of the pixel in the direction of X (decimal value of the degree), 0.00000000000000 - the magnitude of the rotation of the image (always zero), -0.00833333333333 - the negative dimension of the pixel in the direction Y (decimal value degrees), +20.00416666666667 - geographic longitude value for the upper left pixel, +89.99583333333333 - geographic latitude value for the upper left pixel.

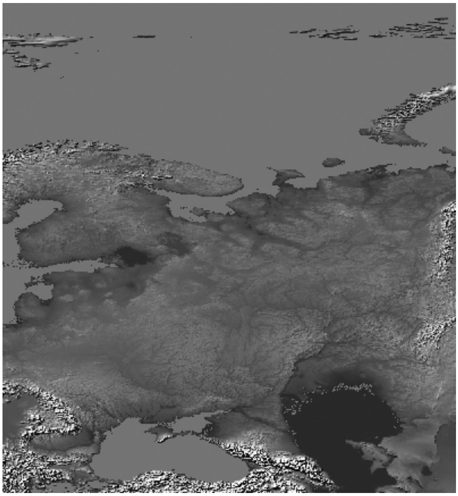


Fig. 2 Relief field in the format SRTM30

The flight profile along the geographical parallel of 49.9974° throughout the region is shown in Fig. 3.

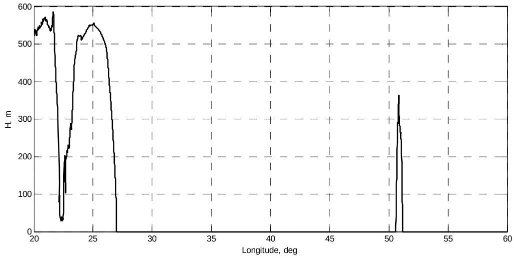
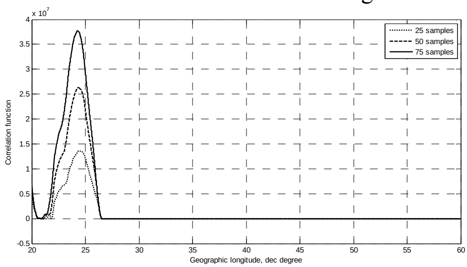
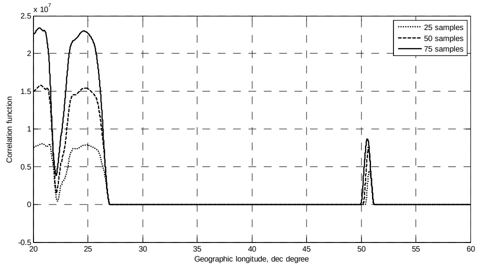


Fig. 3. Flight profile

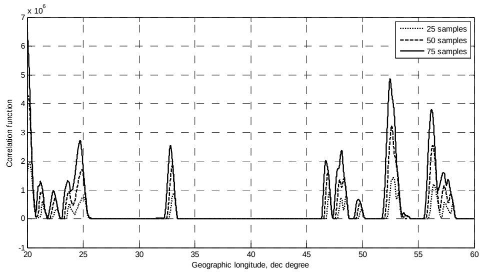
The ideal realization of the field (in the absence of sensor errors) was selected in the direction of movement in varied geographic parallels in the form of 25, 50 and 75 points of the flight profile with a reference point of 24.5875° of eastern longitude. Correlation functions are shown in Fig. 4.



a



b



c

Fig. 4 Correlation functions of current and template field realization for different flight profiles:
a) motion along the parallel 50.2058° of northern latitude with 25, 50 and 75 points of the profile,
b) motion along the parallel along 49.9974° of northern latitude with 25, 50 and 75 points of the profile;
c) motion along the parallel along 49.7891° of northern latitude with 25, 50 and 75 points of the profile;

For the examples under consideration, it is clearly seen that the local search for an optimal solution may result in a stop on the local extremum and a corresponding false matching of the template with the current realization. In addition, the number of measurements in the current realization substantially affects the quality of the correlation function, in particular, with the largest measurement value (75 points), the global extremum is most expressed, but even so, the meaning for the second variant of motion (Figure 4, b, c) is observed to be equivalent extremum, which can also be related to false matching.

Conclusions

In addition, it should be noted that the simulation did not take into account the possible errors of the field sensor, which would also significantly increase the risk of false matching between the template and the current field realization.

Thus, with further consideration, it is necessary to select such CENS mathematical model that will be minimally sensitive to such limiting factors as:

1) multimodal distribution of the probability of comparison of the template and the current field realization (Fig. 4);

2) significant nonlinearity of CENS equations of measurement, in particular, the correspondence between the measured values of field and the object coordinates is tabulated, and in most cases can not be analytically approximated (Fig. 2);

3) the initial uncertainty of the current coordinates due to the increase in time of SINS errors, which significantly affects the area of initial search, and, accordingly, the time and accuracy of the navigation solution.

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