UDC 517.958

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Mathematical modeling of the motion of a mechanical body in a viscous liquid

An arbitrary spatial motion of a body in a viscous non-solvent fluid is investigated. A system of dynamic and kinematic linear differential equations describing this motion is obtained. Asymptotic splitting of the system of differential equations into two subsystems is carried out, one of which describes the longitudinal translational motion of the investigated body, and the other – its lateral motion.

An important tool for studying the dynamic processes of mechanical systems is their mathematical modeling [1]. The set of mathematical relations should adequately and correctly describe the dynamics and statics of the object under study, as well as its behavior and the main characteristics. To study and analyze the complex spatial motion of mechanical bodies in a liquid or gas [2, 3, 4], it is also essential to decompose a complete system of differential equations onto the subsystems that describe the movement of a body in separate directions [5]. It is necessary to take into account a number of features due to the physics of the process, in particular, the nature of aerohydrodynamic forces and moments.

The purpose of this work is to study and simulate the spatial motion of a mechanical object in a non-elastic viscous fluid, as well as the asymptotic splitting of a complete system of linear differential equations describing its motion.

Consider the body that carries out arbitrary spatial movement in a viscous liquid. To simplify the form of the equations describing this motion, we will assume that the body has two planes of symmetry Oxy and Oxz and coordinate axes Ox, Oy, Ox are the main axes of inertia of the body.

Then, due to the symmetry of the body relative to the plane Oxy rotational derivatives $\frac{\partial c_y}{\partial \omega_x} = \frac{\partial c_y}{\partial \omega_y} = \frac{\partial c_z}{\partial \omega_z} = \frac{\partial m_x}{\partial \omega_x} = \frac{\partial m_x}{\partial \omega_y} = 0$, and due to symmetry with respect to the plane Oxz rotational derivatives $\frac{\partial c_z}{\partial \omega_x} = \frac{\partial m_x}{\partial \omega_y} = \frac{\partial m_y}{\partial \omega_x} = 0$.

Assuming that the rotation of the body does not cause a change in the longitudinal force (i.e. $\frac{\partial c_x}{\partial \omega_x} = \frac{\partial c_x}{\partial \omega_y} = \frac{\partial c_x}{\partial \omega_z} = 0$), and using the laws of the amount of motion and moments of the amount of motion [3], we obtain a system of differential equations of motion of a body in a liquid:

$$M\left(1+K_{11}\right)\frac{d\upsilon_x}{dt} - My_c\frac{dw_z}{dt} + Mz_c\frac{dw_y}{dt} + M\left(1+K_{22}\right)\left(\upsilon_z w_y - \upsilon_y w_x\right) -$$
$$-M\,\overline{x}_0\left(w_y^2 + w_z^2\right)L + My_c w_x w_y + Mz_c w_x w_z = T - \frac{1}{2}c_x\rho S\upsilon^2 - P\sin\theta;$$

$$
\begin{aligned}
&M\left(1+K_{22}\right)\frac{d\upsilon_y}{dt}+M\,\overline{x}_0L\frac{dw_x}{dt}+Mx_c\frac{dw_x}{dt}+M\left(1+K_{11}\right)\upsilon_x w_z-\\
&-M\left(1+K_{33}\right)\upsilon_z w_x-My_c\left(w_x^2+w_z^2\right)-Mz_c w_y w_z+M\,\overline{x}_0Lw_x w_y=\\
&=\frac{1}{2}c_y\left(\alpha,\delta_г\right)\rho S\upsilon^2+\frac{1}{2}c_y^{w_z}\rho S\upsilon Lw_x-P\cos\theta\cos\varphi;\\
&M\left(1+K_{33}\right)\frac{d\upsilon_z}{dt}-M\,\overline{x}_0L\frac{dw_y}{dt}+My_c\frac{dw_x}{dt}+M\left(1+K_{22}\right)\upsilon_y w_x-\\
&-M\left(1+K_{11}\right)\upsilon_x w_y-Mz_c\left(w_x^2+w_y^2\right)+M\,\overline{x}_0Lw_x w_z+My_c w_y w_z=\\
&=\frac{1}{2}c_z\left(\beta,\delta_в\right)\rho S\upsilon^2+\frac{1}{2}c_z^{w_y}\rho SL\upsilon w_y+P\cos\theta\sin\varphi;\\
&M\rho_x^2\left(1+K_{44}\right)\frac{dw_x}{dt}+My_c\frac{d\upsilon_x}{dt}-Mz_c\frac{d\upsilon_y}{dt}-My_c\left(\upsilon_x w_y-\upsilon_y w_x\right)+\\
&+Mz_c\left(\upsilon_z w_x-\upsilon_x w_z\right)=\frac{1}{2}m_x\rho SL\upsilon^2+\frac{1}{2}m_x^{w_x}\rho SL^2\upsilon w_x+\frac{1}{2}m_x^{w_y}\rho SL^2\upsilon w_y+\\
&+G\left(z_c\cos\varphi+y_c\sin\varphi\right)\cos\theta;\\
&M\rho_y^2\left(1+K_{55}\right)\frac{dw_y}{dt}-M\,\overline{x}_0L\frac{d\upsilon_z}{dt}-Mz_c\frac{d\upsilon_x}{dt}-M\rho_z^2\left(1+K_{66}\right)w_x w_z+\\
&+M\,\overline{x}_0L\left(\upsilon_x w_y-\upsilon_y w_x\right)+Mz_c\left(\upsilon_z w_y-\upsilon_y w_z\right)=\frac{1}{2}m_y\left(\beta,\delta_в\right)\rho SL\upsilon^2+\\
&+\frac{1}{2}m_y^{w_y}\rho SL^2\upsilon w_y-G\left(z_c\sin\theta+x_c\cos\theta\sin\varphi\right);\\
&M\rho_z^2\left(1+K_{66}\right)\frac{dw_z}{dt}+M\,\overline{x}_0L\frac{d\upsilon_y}{dt}-My_c\frac{d\upsilon_x}{dt}+M\rho_y^2\left(1+K_{55}\right)w_x w_y+\\
&+M\,\overline{x}_0L\left(\upsilon_x w_z-\upsilon_z w_x\right)+My_c\left(\upsilon_y w_z-\upsilon_z w_y\right)=\frac{1}{2}m_z\left(\alpha,\delta_г\right)\rho SL\upsilon^2+\\
&+\frac{1}{2}m_z^{w_z}\rho SL^2\upsilon w_z+G\left(y_c\sin\theta-x_c\cos\theta\cos\varphi\right),
\end{aligned}
\qquad (1)
$$

where: $\upsilon_x, \upsilon_y, \upsilon_z$ – projection of the velocity vector of the translational pole on the corresponding axis of the connected coordinate system; w_x, w_y, w_z – projections of the vector of angular velocity on the corresponding axes of the connected coordinate system; υ – the velocity of the translational motion of the pole; T – traction power; P – residual buoyancy; G – gravity; M – body weight; x_c, y_c, z_c – coordinates of the center of gravity in the axes x, y, z; S – middle area; L – body length; $\delta_г, \delta_в$ – the angles of control of the controls in the horizontal and vertical planes, respectively; ρ – liquid density; α – angle of attack; β – the drift angle;

θ, φ, ψ – Euler angles; ρ_x, ρ_y, ρ_z – radii of inertia; K_{ij} – coefficients of the connected masses;

$$\bar{x}_0 = \bar{x}_c + k_{2\sigma}; \ \bar{x}_c = x_c / L ;$$

$c_z(\beta, \delta_в)$, c_x, $c_y(\alpha, \delta_г)$, $m_z(\alpha, \delta_г)$, $m_y(\beta, \delta_в)$ – coefficients of positional components of hydrodynamic forces and moments.

Let projections of the vector of angular velocity of a body w_x, w_y, w_z are small (we neglect the product of these projections and the sum of their squares), the projection of angular velocity w_y it is much smaller, than projections w_x, w_z, projection of the velocity vector of the translational motion υ_y, υ_z is small compared with the projection υ_x, the deviation of the center of gravity from the diametral plane and the middle plane is small. In order to fully describe the spatial movement of the body, it is necessary to add six kinematic equations to the system of dynamic equations (1). Then a complete system that describes the arbitrary movement of the body in a viscous liquid, has the form

$$\begin{aligned}
&\frac{d\upsilon_x}{dt} = F_{\upsilon_x}\left(\upsilon, \upsilon_x, \theta, \varphi, \alpha, \beta, w_z\right); \quad \frac{d\upsilon_y}{dt} = F_{\upsilon_y}\left(\upsilon, \upsilon_x, \beta, w_z, \theta, \varphi, \alpha\right);\\
&\frac{dw_z}{dt} = F_{w_z}\left(\upsilon, \upsilon_x, \beta, w_z, \theta, \varphi, \alpha\right); \quad \frac{d\theta}{dt} = F_\theta\left(w_z, \varphi\right);\\
&\upsilon_x = \upsilon \cos\beta \cos\alpha; \quad \upsilon_y = -\upsilon \cos\beta \sin\alpha .
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\frac{d\upsilon_z}{dt} = F_{\upsilon_z}\left(\upsilon, \upsilon_x, w_y, \theta, \varphi, \beta, \alpha\right); \quad \frac{dw_x}{dt} = F_{w_x}\left(\upsilon, \upsilon_x, w_x, w_y, \theta, \varphi, \beta, \alpha\right);\\
&\frac{dw_y}{dt} = F_{w_y}\left(\upsilon, \upsilon_x, w_x, w_y, \theta, \varphi, \beta, \alpha\right); \quad \frac{d\psi}{dt} = F_\psi\left(w_y, w_z, \varphi, \theta\right);\\
&\frac{d\varphi}{dt} = F_\varphi\left(w_x, w_y, w_z, \varphi, \theta\right); \quad \upsilon_z = \upsilon \sin\beta;
\end{aligned} \tag{3}$$

$$F_{\upsilon_x} = \frac{A_1}{M(1+K_{11})} + \frac{y_c\left[A_6(1+K_{22}) - (\bar{x}_0 L)A_2\right]}{M(1+K_{11})\left[\rho_z^2(1+K_{22})(1+K_{66}) - (\bar{x}_0 L)^2\right]};$$

$$F_{\upsilon_y} = \frac{A_2\rho_z^2(1+K_{66}) - (\bar{x}_0 L)A_6}{M\left[\rho_z^2(1+K_{22})(1+K_{66}) - (\bar{x}_0 L)^2\right]}; \quad F_{w_z} = \frac{A_6(1+K_{66}) - (\bar{x}_0 L)A_2}{M\left[\rho_z^2(1+K_{22})(1+K_{66}) - (\bar{x}_0 L)^2\right]};$$

$$F_\theta = w_z \cos\varphi; \quad F_{\upsilon_z} = \frac{\rho_y^2(1+K_{55})A_3 + (\bar{x}_0 L)A_5}{M\left[\rho_y^2(1+K_{33})(1+K_{55}) - (\bar{x}_0 L)^2\right]};$$

$$F_{w_x} = \frac{A_4}{M\rho_x^2(1+K_{44})} - \frac{\rho_y^2(1+K_{55})A_3 - (\bar{x}_0 L)A_5}{M\rho_x^2(1+K_{44})\left[(\bar{x}_0 L)^2 + \rho_y^2(1+K_{33})(1+K_{55})\right]};$$

$$F_{w_y} = \frac{(1+K_{33})A_5 + (\bar{x}_0 L)A_3}{M\left[\rho_y^2(1+K_{33})(1+K_{55}) - (\bar{x}_0 L)^2\right]}; \quad F_\psi = \frac{1}{\cos\theta}(w_y\cos\varphi - w_z\sin\varphi);$$

$$F_\varphi = w_x - tg\theta(w_y\cos\varphi - w_z\sin\varphi); \quad A_1 = T - \frac{1}{2}c_x\rho S\upsilon^2 - P\sin\theta;$$

$$A_2 = M(1+K_{11})\upsilon_x w_z + \frac{1}{2}c_y(\alpha,\delta_г)\rho S\upsilon^2 + \frac{1}{2}c_y^{w_z}\rho S\upsilon L w_z - P\cos\theta\cos\varphi;$$

$$A_3 = M(1+K_{11})\upsilon_x w_y + \frac{1}{2}c_z(\beta,\delta_в)\rho S\upsilon^2 + \frac{1}{2}c_z^{w_y}\rho S L\upsilon w_y + P\cos\theta\sin\varphi;$$

$$A_4 = \frac{1}{2}m_x\rho S L\upsilon^2 + \frac{1}{2}m_x^{w_x}\rho S L^2\upsilon w_x + \frac{1}{2}m_x^{w_y}\rho S L^2\upsilon w_y +$$
$$+G(x_c\cos\varphi + y_c\sin\varphi)\cos\theta + My_c\upsilon_x w_y;$$

$$A_5 = \frac{1}{2}m_y(\beta,\delta_в)\rho S L\upsilon^2 + \frac{1}{2}m_y^{w_y}\rho S L^2\upsilon w_y - Gx_c\cos\theta\sin\varphi - M\bar{x}_0 L\upsilon_x w_y;$$

$$A_6 = \frac{1}{2}m_z(\alpha,\delta_г)\rho S L\upsilon^2 + \frac{1}{2}m_z^{w_z}\rho S L^2\upsilon w_z - Gx_c\cos\theta\cos\varphi - M\bar{x}_0 L\upsilon_x w_x.$$

This complete system is represented as two related subsystems (2) and (3). The subsystem (2) describes the longitudinal movement of the body, that is, the translational motion along the axes Ox i Oy, and rotational motion around the axis Oz. The subsystem (3) describes the lateral movement of the body, that is, the motion of the pole along the axis Oz, and the rotation of the body around the axes Ox and Oy.

Let us find out the nature of the dependence of the functions of longitudinal motion $F_{\upsilon_x}, F_{\upsilon_y}, F_{w_z}, F_\theta$ on the variables of lateral motion φ, β, w_y, that is the nature of the dependence of A_1, A_2, A_6 on φ, β, w_y.

Coefficient of the strength of the frontal support c_x is an even function of the variables α, β, but it does not depend on variables w_x, w_y, w_z. Coefficients of positional forces and moments $c_y(\alpha,\delta_г)$ and $m_x(\alpha,\delta_г)$ do not depend on φ, w_x, w_y, w_z. Rotational derivatives $c_y^{w_z}, m_z^{w_z}$ at any angle the attack does not depend on φ, w_x, w_y, w_z, and at small angles of attack (less than 7º) also do not depend on α, β. Given the symmetry of the investigated body derivatives $c_y^{w_z}$ i $m_z^{w_z}$ are even in a variable β. Thus, the function $F_{\upsilon_x}, F_{\upsilon_y}, F_{w_z}$ are even in φ, β [4].

System of equations

$$\upsilon_x = \upsilon\cos\beta\cos\alpha; \quad \upsilon_y = -\upsilon\cos\beta\sin\alpha; \quad \upsilon_z = \upsilon\sin\beta \tag{4}$$

can be easily solved with respect to variables υ, α, β . Substituting

$$\upsilon = f_\upsilon\left(\upsilon_x, \upsilon_y, \upsilon_z\right), \quad \alpha = f_\alpha\left(\upsilon_x, \upsilon_y, \upsilon_z\right), \quad \beta = f_\beta\left(\upsilon_x, \upsilon_y, \upsilon_z\right)$$

in the complete system of equations of motion (2) and (3), we obtain a system of nine differential equations, and the functions in the right-hand sides of the equations of longitudinal motion $F'_{\upsilon_x}, F'_{\upsilon_y}, F'_{w_z}, F'_\theta$, will be like functions $F_{\upsilon_x}, F_{\upsilon_y}, F_{w_z}, F_\theta$ even in variables of lateral movement φ, υ_z, w_y .

Let's introduce the following notation:

$$\eta_1 = \upsilon_x;\ \eta_2 = \upsilon_y;\ \eta_3 = w_z;\ \eta_4 = \theta;\ \eta_5 = \upsilon_z;\ \eta_6 = w_x;\ \eta_7 = w_y;\ \eta_8 = \psi;\ \eta_9 = \varphi;$$

$$F_1 = F_{\upsilon_x}; F_2 = F_{\upsilon_y}; F_3 = F_{w_z}; F_4 = F_\theta; F_5 = F_{\upsilon_z}; F_6 = F_{w_x}; F_7 = F_{w_y}; F_8 = F_\psi; F_9 = F_\varphi.$$

Let us rewrite the system of equations (2), (3) in the vector form

$$\frac{d\eta}{dt} = F\left(t, \eta\right), \tag{5}$$

where $\eta' = \left\|\eta_1, \eta_2, \ldots, \eta_9\right\|; \quad F' = \left\|F_1, F_2, \ldots, F_9\right\|$, «'» – sign of transposition.

Consider the movement of the body in the neighborhood of the nominal motion

$$\eta = \eta_* + \varepsilon\Delta\eta \ . \tag{6}$$

The degree of small perturbation is characterized by some parameter ε , $0 < \varepsilon < 1$. Then

$$\frac{d\eta_*}{dt} + \varepsilon\frac{d\Delta\eta}{dt} = F\left(t, \eta_* + \varepsilon\Delta\eta\right). \tag{7}$$

Let us expand the right part of the relation (7) into a Taylor series by parameter $\varepsilon\,\Delta\eta$:

$$\frac{d\eta_*}{dt} + \varepsilon\frac{d\Delta\eta}{dt} = F\left(t, \eta_*\right) + \varepsilon\left(\sum_\eta \Delta\eta\frac{\partial}{\partial\eta}\right)F_* + \varepsilon^2\left(\sum_\eta \Delta\eta\frac{\partial}{\partial\eta}\right)^2 F_* + \ldots,$$

where

$$\left(\sum_\eta \Delta\eta\frac{\partial}{\partial\eta}\right)F_* = \Delta\eta_1\left(\frac{\partial F}{\partial\eta_1}\right)_* + \Delta\eta_2\left(\frac{\partial F}{\partial\eta_2}\right)_* + \ldots + \Delta\eta_9\left(\frac{\partial F}{\partial\eta_9}\right)_*;$$

$$\left(\sum_\eta \Delta\eta\frac{\partial}{\partial\eta}\right)^2 F_* = \Delta\eta_1^2\left(\frac{\partial^2 F}{\partial\eta_1^2}\right)_* + \Delta\eta_1\Delta\eta_2\left(\frac{\partial^2 F}{\partial\eta_1\partial\eta_2}\right)_* + \ldots$$

Taking into account that identical relations are executed $\dfrac{d\eta_*}{dt} \equiv F\left(t, \eta_*\right)$,

we get $\varepsilon\dfrac{d\Delta\eta}{dt} = \varepsilon\left(\sum_\eta \Delta\eta\dfrac{\partial}{\partial\eta}\right)F_* + \varepsilon^2\left(\sum_\eta \Delta\eta\dfrac{\partial}{\partial\eta}\right)^2 F_* + \ldots$

or $\quad \dfrac{d\Delta\eta}{dt} = \left(\sum_\eta \Delta\eta\dfrac{\partial}{\partial\eta}\right)F_* + \varepsilon\left(\sum_\eta \Delta\eta\dfrac{\partial}{\partial\eta}\right)^2 F_* + \ldots$

Each of the equations will be written in detail as follows:

$$\begin{aligned}
\frac{d\Delta\upsilon_x}{dt} &= \left(\Delta\upsilon_x \frac{\partial F_{\upsilon_x}}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial F_{\upsilon_x}}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial F_{\upsilon_x}}{\partial \upsilon_z} + \Delta w_z \frac{\partial F_{\upsilon_x}}{\partial w_z} + \Delta\theta \frac{\partial F_{\upsilon_x}}{\partial \theta} + \Delta\varphi \frac{\partial F_{\upsilon_x}}{\partial \varphi} \right) + \\
&+ \varepsilon \left(\Delta\upsilon_x \frac{\partial}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial}{\partial \upsilon_z} + \Delta w_z \frac{\partial}{\partial w_z} + \Delta\theta \frac{\partial}{\partial \theta} + \Delta\varphi \frac{\partial}{\partial \varphi} \right)^2 F_{\upsilon_x} + \ldots; \\
\frac{d\Delta\upsilon_y}{dt} &= \left(\Delta\upsilon_x \frac{\partial F_{\upsilon_y}}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial F_{\upsilon_y}}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial F_{\upsilon_y}}{\partial \upsilon_z} + \Delta w_z \frac{\partial F_{\upsilon_y}}{\partial w_z} + \Delta\theta \frac{\partial F_{\upsilon_y}}{\partial \theta} + \Delta\varphi \frac{\partial F_{\upsilon_y}}{\partial \varphi} \right) + \\
&+ \varepsilon \left(\Delta\upsilon_x \frac{\partial}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial}{\partial \upsilon_z} + \Delta w_z \frac{\partial}{\partial w_z} + \Delta\theta \frac{\partial}{\partial \theta} + \Delta\varphi \frac{\partial}{\partial \varphi} \right)^2 F_{\upsilon_y} + \ldots; \\
\frac{d\Delta w_z}{dt} &= \left(\Delta\upsilon_x \frac{\partial F_{w_z}}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial F_{w_z}}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial F_{w_z}}{\partial \upsilon_z} + \Delta w_z \frac{\partial F_{w_z}}{\partial w_z} + \Delta\theta \frac{\partial F_{w_z}}{\partial \theta} + \Delta\varphi \frac{\partial F_{w_z}}{\partial \varphi} \right) + \\
&+ \varepsilon \left(\Delta\upsilon_x \frac{\partial}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial}{\partial \upsilon_z} + \Delta w_z \frac{\partial}{\partial w_z} + \Delta\theta \frac{\partial}{\partial \theta} + \Delta\varphi \frac{\partial}{\partial \varphi} \right)^2 F_{w_z} + \ldots; \\
\frac{d\Delta\theta}{dt} &= \left(\Delta w_z \frac{\partial F_\theta}{\partial w_z} + \Delta\varphi \frac{\partial F_\theta}{\partial \varphi} \right) + \varepsilon \left(\Delta w_z \frac{\partial}{\partial w_z} + \Delta\varphi \frac{\partial}{\partial \varphi} \right)^2 F_\theta + \ldots .
\end{aligned} \tag{8}$$

$$\begin{aligned}
\frac{d\Delta\upsilon_z}{dt} &= \left(\Delta\upsilon_x \frac{\partial F_{\upsilon_z}}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial F_{\upsilon_z}}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial F_{\upsilon_z}}{\partial \upsilon_z} + \Delta w_y \frac{\partial F_{\upsilon_z}}{\partial w_y} + \Delta\theta \frac{\partial F_{\upsilon_z}}{\partial \theta} + \Delta\varphi \frac{\partial F_{\upsilon_z}}{\partial \varphi} \right) + \\
&+ \varepsilon \left(\Delta\upsilon_x \frac{\partial}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial}{\partial \upsilon_z} + \Delta w_y \frac{\partial}{\partial w_y} + \Delta\theta \frac{\partial}{\partial \theta} + \Delta\varphi \frac{\partial}{\partial \varphi} \right)^2 F_{\upsilon_z} + \ldots; \\
\frac{d\Delta w_x}{dt} &= \left(\Delta\upsilon_x \frac{\partial F_{w_x}}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial F_{w_x}}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial F_{w_x}}{\partial \upsilon_z} + \Delta w_x \frac{\partial F_{w_x}}{\partial w_x} + \Delta w_y \frac{\partial F_{w_x}}{\partial w_y} + \Delta\theta \frac{\partial F_{w_x}}{\partial \theta} + \Delta\varphi \frac{\partial F_{w_x}}{\partial \varphi} \right) + \\
&+ \varepsilon \left(\Delta\upsilon_x \frac{\partial}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial}{\partial \upsilon_z} + \Delta w_x \frac{\partial}{\partial w_x} + \Delta w_y \frac{\partial}{\partial w_y} + \Delta\theta \frac{\partial}{\partial \theta} + \Delta\varphi \frac{\partial}{\partial \varphi} \right)^2 F_{w_x} + \ldots; \\
\frac{d\Delta w_y}{dt} &= \left(\Delta\upsilon_x \frac{\partial F_{w_y}}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial F_{w_y}}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial F_{w_y}}{\partial \upsilon_z} + \Delta w_y \frac{\partial F_{w_y}}{\partial w_y} + \Delta\theta \frac{\partial F_{w_y}}{\partial \theta} + \Delta\varphi \frac{\partial F_{w_y}}{\partial \varphi} \right) + \\
&+ \varepsilon \left(\Delta\upsilon_x \frac{\partial}{\partial \upsilon_x} + \Delta\upsilon_y \frac{\partial}{\partial \upsilon_y} + \Delta\upsilon_z \frac{\partial}{\partial \upsilon_z} + \Delta w_y \frac{\partial}{\partial w_y} + \Delta\theta \frac{\partial}{\partial \theta} + \Delta\varphi \frac{\partial}{\partial \varphi} \right)^2 F_{w_y} + \ldots; \\
\frac{d\Delta\psi}{dt} &= \left(\Delta w_y \frac{\partial F_\psi}{\partial w_y} + \Delta w_z \frac{\partial F_\psi}{\partial w_z} + \Delta\varphi \frac{\partial F_\psi}{\partial \varphi} + \Delta\theta \frac{\partial F_\psi}{\partial \theta} \right) +
\end{aligned} \tag{9}$$

$$+\varepsilon\left(\Delta w_y\frac{\partial}{\partial w_y}+\Delta w_z\frac{\partial}{\partial w_z}+\Delta\varphi\frac{\partial}{\partial\varphi}+\Delta\theta\frac{\partial}{\partial\theta}\right)^2 F_\psi+...;$$

$$\frac{d\Delta\varphi}{dt}=\left(\Delta w_x\frac{\partial F_\varphi}{\partial w_x}+\Delta w_y\frac{\partial F_\varphi}{\partial w_y}+\Delta w_z\frac{\partial F_\varphi}{\partial w_z}+\Delta\varphi\frac{\partial F_\varphi}{\partial\varphi}+\Delta\theta\frac{\partial F_\varphi}{\partial\theta}\right)+$$

$$+\varepsilon\left(\Delta w_x\frac{\partial}{\partial w_x}+\Delta w_y\frac{\partial}{\partial w_y}+\Delta w_z\frac{\partial}{\partial w_z}+\Delta\varphi\frac{\partial}{\partial\varphi}+\Delta\theta\frac{\partial}{\partial\theta}\right)^2 F_\varphi+...$$

Given that $F_{\upsilon_x}, F_{\upsilon_y}, F_{w_z}$ are even in υ_z, φ, and F_θ is even in φ, in the Taylor expansion there are no members

$$\frac{dF_{\upsilon_x}}{d\varphi}=\frac{dF_{\upsilon_x}}{d\upsilon_z}=\frac{dF_{\upsilon_y}}{d\varphi}=\frac{dF_{\upsilon_y}}{d\upsilon_z}=\frac{dF_{w_z}}{d\varphi}=\frac{dF_{w_z}}{d\upsilon_z}=\frac{dF_\theta}{d\varphi}=0,$$

that is subsystem (8) with $\varepsilon=0$ (in the linear approximation) integrates regardless of the system (9).

As a result of the study of the motion of the body in a viscous non-resistible liquid, a complete system of differential equations describing the dynamics of this body is obtained. The complex motion of the body was considered as a combination of longitudinal and lateral movements, and these movements are interrelated, since the variables of the longitudinal motion $\upsilon, \upsilon_x, \upsilon_y, w_z, \theta, \alpha$ and alternating lateral movements $\upsilon_z, w_x, w_y, \psi, \varphi, \beta$ are included in both subsystems. In the linear approximation, the longitudinal movement of the body, that is, the translational motion along the axes Ox and Oy and rotational motion around the axis Oz, does not depend on the lateral movement, but the lateral movement, that is, the motion of the pole along the axis Oz and the rotation of the body around the axes Ox and Oy, depends on the longitudinal motion.

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