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Inertial Navigation Accuracy and Accelerometers with damping

The problem of inertial navigation accuracy with using of accelerometers in inertial measurement unit are discussed. It is shown that a strict successive analysis of accelerometer measurement procedure makes it possible to understand probable new source of errors in the value estimation of the disturbing force magnitude. If do not take into account these new possible errors any self-adjusting system of an airplane may be functionally unreliable. These errors arise only when temporal duration of disturbing force is less than some critical time τ, which depends on the given characteristics of an aircraft Inertial Navigation System and aircraft itself. The method of finding τ is proposed. A presence of a damping force in the proof mass motion equation is examined.

Introduction

The navigation accuracy depends on probable sources of errors in the value estimation of the disturbing force magnitude. The problem of navigation accuracy whose roots lay back in the mid-twentieth century should be extended now to meet the new challenges. Amongst these well-known challenges we would mention enormously high frequency of flights, continuously disproving environmental conditions, problems with airport landing and we should place in the first position the terrorist incidents in civil aviation which include aircraft hijacking, airlines bombing, terrorist attacks on airports. Common future of all above mentioned items is the extremely short segment of time available for checking out the health of the avionic system on the flight line or on the ground. Namely, this common point makes it possible to gather consideration of all these distinct situations within one theoretical framework and elaborate mathematical tools to construct admissible diagnostic procedures. Amongst all novel up-to-date challenges we should also mention the perspective of integrated commercial satellite system development of aircraft of civil aviation [1] and the creation of the low-cost gyro-free inertial navigation system (INS) [2]. Some new approach to elaborate theoretical framework to construct optimal flight safety diagnostic procedures was initiated in [3]. If the navigation schemes don't change they can become extinct, clearly, modern aircraft navigation system are being put at risk by the electronic devices that passengers carry on board, such as laptop computers and similar devices. Because of plethora of as mentioned and omitted anxiously important problems we should restrict ourselves to the analysis the most widespread inescapable external disturbing influences which of them differs of each other in the temporal duration. In this connection a presence of a damping force in the proof mass motion equation is examined.

Accelerometers and Influence Duration

The main part of INS is so called Inertial Measurement Unit (IMU). INS play anxiously important role in the modern aviation. The problem of INS construction dates from the early 20th century. The methods of INS continue to be an area of active research and from time to time had to be extended to meet the new challenge [1] – [11].

This chapter provides a brief introduction to navigation using inertial sensors, explaining only the underlining principles. Inertial sensors comprise accelerometers, which measure specific force and gyroscopes commonly abbreviated to gyros, which measure angular rate. An inertial measurement unit (IMU) combines multiple accelerometers and gyros, usually three of each, to produce three-dimensional measurements of specific forces and angular rate. By integrating these measurements and applying a gravity model, a position, velocity, and attitude solution may be maintained, a concept known as inertial navigation. Practical inertial navigation systems have been available from the 1950s, but were initially very large and expensive. In early INS, the sensors were physically aligned with the horizontal and vertical by mounting them on a platform connected to the host body by a series of gimbals driven by motors. This was known as a platform configuration and was due to the limitations of early gyro technology and the need to minimize the computational load. The strap down configuration, whereby the sensors are aligned with the host body, was first proposed in 1962 with production of the first aircraft systems starting at the end of the 1970s. Today, it is almost universal [9].

Recently, in many countries the efforts of researches are directed to the creation of the low-cost gyro-free INS using linear accelerometers only for navigation of various vehicles[2]. So, in this chapter we restrict ourselves primarily to the analysis of accelerometers only. Let us shortly consider a simple method to measure probable disturbing influences on a steady flying motion of an airplane [6].

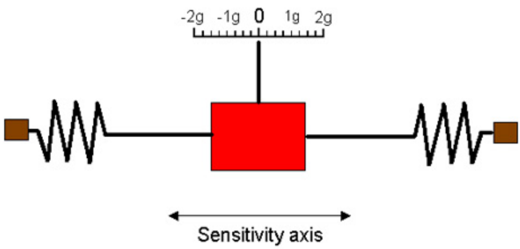

Fig. 1. Accelerometer principle

By attaching a mass to a spring, measuring its deflection, we get a simple accelerometer. An important exception is gravitational acceleration. This acts on the proof mass directly, not through the springs, and applies the same acceleration to all components of the accelerometer, so there is no relative motion of the mass with respect to the case. Therefore, the accelerometers sense only the nongravitational acceleration, known as specific force. Using three (or more) accelerometers we can form a 3D specific force measurement f_{IB}^{B}.

The previous consideration shows the extremely important role of damping forces in the realization of any practically significant scheme to construct an accelerometer. Strange as it seems but a presence of a damping force in the proof mass motion equation hides possibilities to discover some new sources of possible errors in disturbing force magnitude3 evaluation. So, this chapter purposely ignoring damping provides a strict successive analysis of accelerometer measurement procedure. Such a consideration makes it possible to understand probable source of errors in the value estimation of the disturbing force magnitude. The consequences of such errors may be anxiously substantial, so that can at some circumstances absolutely destroy the reliability any self-adjusting system of an airplane. Let consider a standard calculation schemes which is a starting point to elaborate their proper self-adjusting system. In the any local inertial frame (LIF) the total disturbing external force F^{ext} according to the center-of-mass theorem gives birth to the acceleration a_c of airplane

$$a_c = \frac{F^{\text{ext}}}{M}, \tag{2}$$

where, M is airplane instant total mass which includes masses of all internal devices and instant amount of fuel. Of course, it is trivial approximate result ignoring after all the continuous descent of fuel. In the noninertial frame rigidly connected with accelerated vehicle (airplane or spacecraft) resulting instant net force $F(t)$ acting on accelerometer proof mass m according to selected direction (Fig. 1) maybe write down as:

$$F(t) = -k_1 \Delta l_1(t) + k_2 \Delta l_2(t) - m a_c\,, \tag{3}$$

where $k_1, \Delta l_1(t)$, stand accordingly for elasticity factor and a change in length of right spring, index “2” means the left spring. For simplicity we set $k_1 = k_2 = k$. Under condition $l_1 + l_2 = l = \text{const}$ we have $dl_1 = -dl_2$, so $\Delta l_1 = -\Delta l_2;\ |\Delta l_1| = |\Delta l_2| = \Delta l$ substitution of these relation and (1) into (2), in the case when $F(t) = 0$ gives

$$m\frac{F^{\text{ext}}}{M} = 2\kappa \Delta l, \tag{4}$$

So,

$$F^{\text{ext}} = \kappa \Delta l, \tag{5}$$

where, $\kappa = \dfrac{2kM}{m}$ so called accelerometer factor.

But (4) is improperly derived formula – absence of forces means absence of accelerations but not of velocities. Moreover, (4) ignores time-dependence of Δl and it is crucial in the case when the duration of influence is less that the time segment between zeroth and maximum value of Δl. Proper calculations can be realized using the work-energy theorem, also known as kinetic energy increase theorem, which states that the work done by all forces (including fictitious one) acting on a particle equals the change in the particles kinetic energy. It is customary in theoretical physics to denote kinetic energy as *T*, so change in the kinetic energy should be denoted as ΔT. How accelerometers proof mass approaches to its halt? How its position depends on time? The answers on these questions may describe correlation between duration of influence and data for self-adjusting system and somehow avoid extremely high errors in the estimation of the disturbing force. To answer on these questions we should use the above mentioned theorem and introduce oriented along motion (Fig. 1) coordinate axis *x,* which origin coincides with initial rest-point of accelerometer mass *m.* So, we can wright down

$$\Delta T = \int_0^{\Delta l(t)} (2k\Delta l(t) - ma_c)dx \, , \tag{6}$$

Suppose that accelerometer mass approaches to its halt at time $\Delta t = \tau$ which means $\Delta T = 0,$ so (5) may be rewritten as

$$2\int_0^{\Delta l(t)} k\Delta l(t)dx - m\int_0^{\Delta l(t)} ma_c dx = 0 \, , \tag{7}$$

After integration using all above mentioned relations we get simple equation

$$2k\left(\Delta l(t)\right)^2 - ma_c\Delta l(\tau) = 0 \, , \tag{8}$$

which has two solutions: first $\Delta l(\tau) = 0$ that means the initial position, when disturbing forces begin to act and second

$$\Delta l(\tau) = \frac{ma_c}{2k} = \frac{mF^{\text{ext}}}{2Mk}, \tag{9}$$

from which we exactly derive relation (4), noting that $\Delta l(\tau) = 2\Delta l.$

Suppose now that time segment Δt of disturbing force F^{ext} action is less than τ , i.e. $\Delta t < \tau.$ In this case using in all existing IMU schemes relation (4) is incorrect and we deal with enormously large errors in evaluating of F^{ext} . So, we should return to the equation (5) in which at this time $\Delta T \neq 0$ and we have opportune y to find answer the questions what should be after the moment of a disappearance of a disturbing force F^{ext} and how we can properly evaluate the F^{ext} in this case. After the moment when external influence become extinct accelerometer proof mass *m* continues to move. In accelerated frame Lagrange function *L* has the form [13]

$$L = \frac{m\dot{x}^2}{2} - ma_c x - U \, , \tag{10}$$

where, $U = \dfrac{mx^2}{2}$ and proper equations of motion, which correspond to Lagrange function (9) are integrating in general form. At that we should not even write down original motion equation and can start right away from its first integral *E*. So, for Lagrange function (9) we have

$$E = \frac{m\dot{x}^2}{2} + ma_c x + U\,, \tag{11}$$

This is the first order differential equation. Integration yields

$$\frac{dx}{dt} = \sqrt{\frac{2}{m}[E - U(x) - ma_c x}\,, \tag{12}$$

after integration we obtain for *t*

$$t = \sqrt{\frac{m}{2}} \int \frac{dx}{\sqrt{E - U(x) - ma_c}} + \tau_0. \tag{13}$$

Integration constants *E* and τ_0 play role of fitting parameters, which allow apply this quite general framework to every given real data. A simple consideration shows that at such circumstances the maximum value of Δl should be observed in the case of continuous presence of disturbing force, which can be evaluated by means of formula (4). Consequences of this consideration provide unexpected result: shorter then τ temporal duration of disturbing force may be estimated as less in magnitude than real acting force.

Conclusion

A strict successive analysis of accelerometer measurement procedure makes it possible to understand probable new source of errors in the value estimation of the disturbing force magnitude. The consequences of such errors may be at some circumstances anxiously substantial. If don't take into account these new possible errors any self-adjusting system of an airplane may be functionally damaged or become almost unreliable This errors arise only when temporal duration of disturbing force is less than some critical time τ, which depends on some characteristics of an aircraft INS, and aircraft itself. The method of finding τ is given. This conclusion is valid only for influences shorter or equal to some critical duration which can be evaluated by means of formula (4). Further descend of disturbing duration makes all above discussed INS methods to be extinct, and influences on aircrafts should be described by means of 3D wave equation and its solutions, known as plane waves. For the investigation of these problems and also for the consideration of the extended (to involve damping) previous analysis will be devoted future publications.

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