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Y. Krashanytsya, doctor of science,
A. Hoshmandi, aspirant, N. Hristenko, aspirant,
(National Aerospace University by it. N.E.Joukowsky «KHAI»,Kharkov, Ukraine)

Implementation of the method of boundary integral equations in boundary value problems of gasdynamics

Results of generalizations of the original vector and tensor analysis and its applications in terms of the method of boundary integral equations and its numerical implementation for solving nonlinear boundary value problems of hydro- and geodynamics are presented. Fundamental advantage of this method is that the presented integral equalizations, by virtue of the know scope terms, are linear, unlike the differential forms of models.

The most reliable and proven mathematical model of a viscous fluid is the initial-boundary value problem in the form of a system of differential equations in partial, first published in the modern form by J.C. Maxwell, G. G. Stokes and C. L. Navier.

Finding solution to the Navier-Stokes initial-boundary value problem for a system of differential equations in partial derivatives is an important and challenging task in applied mathematics and mechanics; and its solution will significantly change the way of the hydro- and aerodynamic calculations are conducted; improve the quality of the calculations and increase the reliability of the results.

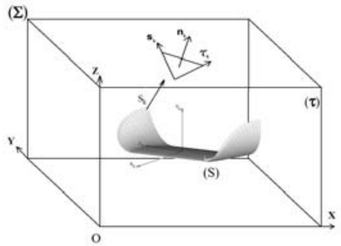

Figure 1. Fixed wing (S) in a steady flow of a viscous compressible fluid inside the control volume (Σ)

The boundary integral approach has obvious advantages over the finite difference and finite element methods. The boundary-integral method can be successfully applied to solution of complex engineering problems – on surface and in space, stationary and time-dependent.

The stationary problem [1] for the flow of the compressible viscous fluid around a body is shown in Fig. 1. The most effective method for solving a wide spectrum of boundary value problems of continuum mechanics is the method of boundary integral equations [2]. In the absence of internal moments and temperature effects, a mathematical model of the dynamics of an incompressible fluid flow is described using a well-known system of conservation laws:

– mass $$(\nabla, \rho \mathbf{V}) = 0, \qquad (1)$$

– momentum $$(\nabla, \mathbf{P}) = 0, \qquad (2)$$

– energy $$(\nabla, \mathbf{E}) = 0, \qquad (3)$$

where tensor $\mathbf{P} = \rho\mathbf{VV} + \mathbf{Й}\left(p - \frac{2}{3}\frac{\mu}{\text{Re}}(\nabla,\mathbf{V})\right) - 2\frac{\mu}{\text{Re}}\nabla^{*}\mathbf{V} + \frac{\mu}{\text{Re}}[\mathbf{ЙЩ}]$, vector of energy $\mathbf{E} = (E + \text{p})\mathbf{V} + (\mathbf{\Phi},\mathbf{V})$,$\mathbf{V}$ is vector of velocity of fluid flow; $\nabla^{*}\mathbf{V}$ is the dual tensor $\nabla\mathbf{V}$, ρ is density of the medium, p is pressure, ν is kinematical viscosity and **I** is an identity tensor. Solution of the system of differential equations of the conservation laws (1 - 3) is subject to the natural boundary condition

$$\mathbf{V}\big|_S(s,\tau) = 0;\ \mathbf{V}\big|_\Sigma(s,\tau) = \mathbf{U}\big|_\infty,\ p\big|_\Sigma = p_\infty,\ \rho\big|_\Sigma = \rho_\infty,\ \mathbf{Щ}_n\big|_S = 0,\ \mathbf{Щ}\big|_\Sigma = 0, \quad (4)$$

where $\mathbf{V}\big|_S$ is the velocity of points on the liquid surface of the body and is a boundary condition that may depend on the surface coordinates (ξ,η) in some particular cases.

Integrating in space (Σ) a combination of differential operators (1 - 3) and taking the standard limit, and also taking into account the properties of the double-layer potential [3], the fundamental solution of Laplace's equation in R^3, we have an integral representation solutions

$$c\mathbf{V} = -\oiint_{(\partial E)} \left\{ 2\left(\left(\frac{\boldsymbol{\phi}}{H_n}\frac{\partial v_\phi}{\partial n} + \frac{\mathbf{s}}{H_n}\frac{\partial v_s}{\partial n}\right),\boldsymbol{\Gamma}\right) + \left(\left(\mathbf{V},[\mathbf{n},[\nabla,\boldsymbol{\Gamma}]]\right) + \left(\mathbf{V},\frac{\partial\boldsymbol{\Gamma}}{\partial n}\right)\right)\right\} c\,dS, \quad (5)$$

$$\mathbf{P} = \oiint_{(\partial E)} \left\{ \left(\left(\mathbf{n},[\nabla,[\mathbf{ЙP}]]\right),\boldsymbol{\Gamma}\right) - \left(\left(\mathbf{P},[\mathbf{n},[\nabla,\boldsymbol{\Gamma}]]\right) + \left(\mathbf{P},\frac{\partial\boldsymbol{\Gamma}}{\partial n}\right)\right)\right\} dS, \quad (6)$$

$$\mathbf{E} = \oiint_{(\partial E)} \left\{ \left(\left([\mathbf{n},[\nabla,\mathbf{E}]] + \left(\frac{\partial\mathbf{E}}{\partial n}\right)\right),\boldsymbol{\Gamma}\right) - \left(\left(\mathbf{E},[\mathbf{n},[\nabla,\boldsymbol{\Gamma}]]\right) + \left(\mathbf{E},\frac{\partial\boldsymbol{\Gamma}}{\partial n}\right)\right)\right\} dS \quad (7)$$

where the tensor $\boldsymbol{\Gamma} = \mathbf{I}\varphi - [\mathbf{I},\mathbf{G}]$, $\varphi(|\mathbf{x}-\mathbf{y}|)$, is the fundamental solution of Laplace's equation in R^3, and the vector $\mathbf{G} \in C^2(E)$ defined by the conservative condition

$$(\nabla,\boldsymbol{\Gamma}) = 0 \Leftrightarrow \nabla\varphi = [\nabla,\mathbf{G}] \quad (8)$$

is the fundamental solution of the differential operators in $\nabla(\nabla,\mathbf{a}) = 0$, i.e.

$$\nabla(\nabla,\boldsymbol{\Gamma}) = \mathbf{I}\delta(|\mathbf{x}-\mathbf{y}|), \quad (7)$$

where $\delta(|\mathbf{x}-\mathbf{y}|)$ is the Dirac delta function, which depends on two points in space, and the vectors and tensors takes place of any of the kinematic characteristics of the problem.

Some results of numerical decision of task of flowing around of wing of eventual scope of type are below presented by the “winglet” stream of viscid gas by the developed method of border integral equalizations and their comparing to the authorial experiment [5]. Numerical solution of linear systems of boundary integral equations type (5 – 7) for the kinematics and dynamic properties can be obtained using the classical quadrature integration over each element of the triangulated surface [3]. As a result of numerical experiments obtained both distributed and total hydrodynamic characteristics. These results [4] are illustrated the wide range of capabilities of the developed method in terms of computing distributed and total hydrogasdynamic characteristics of flow around bodies of arbitrary spatial configuration subjected to the flow and external forces.

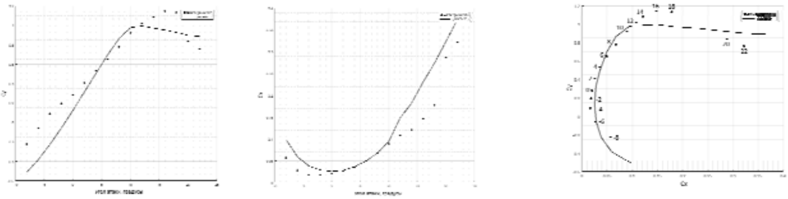

Figure 2. The total aerodynamic characteristics of the wing with endings of the “winglet” type calculated with the experimental number Re=3◦10^4.

Conclusion

The results of the numerical implementation of the method of boundary integral equations on the basis of common concepts of integrated solutions (5 – 7) in order to determine the distribution and total kinematic and dynamic aerodynamic characteristics of planar and spatial elements of bearing systems of aircraft widely used. Fundamental advantage of this method is that the presented integral equalizations, by virtue of known scope terms, are linear unlike the differential forms of models.

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