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D.P. Kucherov, DoS
(National Aviation University, Ukraine)

Backstepping algorithm for controlling non-linear objects

One control algorithm for non-linear objects is considered, based on the introduction of an auxiliary nonlinear function and referring it's to the input of the system through the integral, obtaining its derivative. The algorithm provides global stability of dynamic systems with respect to the endpoint, for example, the origin of coordinates when a system is considered in the phase plane. An example of the application of this algorithm is given.

Introduction

The widespread use of unmanned aerial vehicles presented new challenges for control system engineers. This is due to the peculiarities of these machines since they have not only a complex nonlinear dynamic but also are mathematically described by systems of nonlinear differential equations [1]. The need to ensure the steady movement of such a machine leads to an increase in the number of autonomously controlled control channels (rotors) to achieve the required type of motion of the control object even in the event of failure of one of the channels. Not the last factor is the cost of development, which can be significantly lower in conditions of unification of the construction of the full model.

Traditional approaches, based on rigid feedback or optimization techniques, do not allow to fully provide the required quality of management. Recently, the direction of the construction of feedback systems is developing, ensuring the satisfaction of a certain Lyapunov type function or its derivative, which guarantees the boundedness of the trajectory and the convergence of the final state to a certain one, called stable [2-5]. This type of control makes it possible to provide a robust stabilization of the trajectory of motion and even to provide control under conditions of parametric uncertainty, even if the upper limit is completely undefined. This is provided by the technique of backstepping.

Backstepping is a recursive method for controlling a nonlinear object covered by a rigid feedback, which ensures the stability of the system by the Lyapunov type criterion. In this case, the structure of the system, described by matrix differential equations, acquires the lower triangular form. Next, the backstepping algorithm is considered.

The main idea of the algorithm

Consider a dynamical system described by differential equations of the form

$$\dot{x}(t) = f(x,t) + g(x,t)\pi(t), \tag{1}$$

$$\dot{\pi}(t) = u, \tag{2}$$

in which x, u are vectors such that $x \in R^n$, $u \in R$, where n is an integer and $f(\cdot)$, $g(\cdot)$ are known smooth functions of nonlinear form, the variable t is a time. The control action u can be as a piecewise-continuous as a smooth function, depending also on the states $x(t)$, i.e. $u(t) = \mu(x, t)$ depends on the states $x(t)$, which ensures stabilization of the current position of the system. The aim of the control action is to set the values $x \to 0$, $\pi \to 0$ for $t \to \infty$.

It is also assumed that there is a continuously differentiable function $V(x, t)$ of Lyapunov type satisfying an inequality of the form

$$\frac{\partial V}{\partial x}(f(x,t) + g(x,t)\mu(t)) \leq -W(x), \tag{3}$$

where $W(x)$ is a positive-definite function. In (1) we add and subtract the summand $g(x)\mu(x)$, we obtain

$$\dot{x} = f(x) + g(x)\mu(x) + g(x)[\pi - \mu(x)]. \tag{4}$$

In the expression (4) we denote $e_u = \pi - \mu(x)$, then we get

$$\dot{x} = f(x) + g(x)\mu(x) + g(x)e_u. \tag{5}$$

$$\dot{e}_u = u - \dot{\mu}(x). \tag{6}$$

If the functions $f(\cdot)$, $g(\cdot)$ and $\pi(\cdot)$ are known, then the derivative

$$\dot{\mu} = \frac{\partial \mu}{\partial x}[f(x) + g(x)\pi]. \tag{7}$$

If we synthesize the pseudo-control $\mu(x) \to \pi$, then for $t \to \infty$, we obtain a dynamical system equivalent to (1) except for the state where the input is zero. Thus, the aim of the synthesis is the development of an additional stabilizing control action $v(x)$ that satisfies the original system, i.e.

$$v = u - \dot{\mu}(x). \tag{8}$$

Let us now consider the system (1) with control (8) and determine the stabilization conditions for the trajectory with control of the selected type

$$\dot{x} = f(x) + g(x)\mu(x) + g(x)e_u. \tag{9}$$

$$\dot{e}_u = v. \tag{10}$$

We consider as a function $V(x, \mu)$ for example such as

$$V(x,\mu) = V(x) + \frac{1}{2}\dot{e}_u. \tag{11}$$

Now we calculate the derivative of the function $V(x)$ along the trajectory of the dynamical system (5), (6) and evaluate it

$$\dot{V} = \frac{\partial V}{\partial x}(f + g\mu) + \frac{\partial V}{\partial x} g e_u + e_u v \leq -W(x) + \frac{\partial V}{\partial x} g e_u + e_u v. \tag{12}$$

To simplify the writing, the arguments of the functions used in expression (12) are omitted.

The control function is selected in the form

$$v = -\frac{\partial V}{\partial x} g(x) - k e_u, \tag{13}$$

where $k > 0$.

Substituting (13) into (12) allows us to obtain an expression for the derivative of the function V

$$\dot{V} \leq -W(x) - k e_u^2, \tag{14}$$

which shows that the endpoint ($x = 0$, $e_u = 0$) is asymptotically stable. Then the control law for the dynamical system (1), (2) acquires the form

$$u = v + \dot{\mu}(x) = \frac{\partial \mu}{\partial x}[f(x) + g(x)\pi] - \frac{\partial V}{\partial x} g(x) - k\big(\pi - \mu(x)\big). \tag{15}$$

Example

Consider a second-order system of the form

$$\dot{x}_1 = x_1 + x_1^2 + x_{2,} \tag{16}$$

$$\dot{x}_2 = u. \tag{17}$$

The task is to stabilize the final position, i.e. provide $x_1 \to 0$, $x_2 \to 0$ by the synthesized control u. Now we use the backstepping algorithm.

We put in correspondence to the above algorithm the functions of system (16), (17), i.e. $f = x_1 + x_1^2$, $g = 1$. We introduce the function $\mu(x_1)$ such that $\mu(0)=0$, for example, $\mu(x_1) = -x_1 - x_1^2 - x_1^3$, and the function $V(x_1) = \frac{1}{2}x_1^2$. Then $\dot{x}_1 = -x_1^3$, and

$$\dot{V} = -x_1^4 \le -x_1^2. \quad (18)$$

Inequality (18) holds for all $x_1 \in R$. The stabilizing control has the form

$$u = \frac{\partial\mu}{\partial x_1}\left(x_1 + x_1^2 + x_{2,}\right) - \frac{\partial V}{\partial x_1} - (x_2 + x_1 + x_1^2 + x_1^3) = (2x_1 + 1)\left(x_1 + x_1^2 + x_{2,}\right) - x_1 - (x_2 + x_1 + x_1^2 + x_1^3) \quad (19)$$

The control law (19) provides global stabilization of the control system (16), (17) in the origin region, i.e. when $x_1 = 0$, $x_2 = 0$.

The criterion for stabilizing the system is a function $V(x)$ of the form

$$V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}(x_2 + x_1 + x_1^2 + x_1^3)^2. \quad (20)$$

Conclusions

The algorithm considered allows us to find functions that ensure global stabilization of nonlinear dynamical systems. The required functions are a quadratic type and therefore are positively defined in the range of all values accepted by the state variables. The main idea of the algorithm is to eliminate unwanted signal components by introducing an auxiliary function and assigning it via the integral, i.e. bringing it to the derivative (step back to the input). Further research is planned to focus on the development of practical systems.

References

1. Kucherov D.P., Kozub A.N. Mathematical model of UAV as agent of multi-agent system. *Proc. of the 2005 IEEE International Conference on Dependable Systems, Services, and Technologies* (DESSERT-2018), May 24-27, 2018, Kyiv, Ukraine, p.358-362.
2. Sheng Z., Wei-qi Q. Dynamic backstepping control for pure-feedback nonlinear systems. Access to file: https://arxiv.org/abs/1706.08641.
3. Khalil H.K., *Nonlinear Systems* (Prentice Hall, New York, 2002)
4. Krstic M., Kanellakopoulos I., Kokotovic P.V., *Nonlinear and Adaptive Control Design* (Wiley–Interscience, New York, 1995).
5. Bouabdallah S., Siegwart R. Backstepping and Sliding-mode Techniques Applied to an Indoor Micro Quadrotor. *Proc. of the 2005 IEEE International Conference on Robotics and Automation*, Barcelona, Spain, April 2005, p. 2247-2252.