Investigation of the influence of teeth correction of worm gear with involute worm on contact parameters, wear and life

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Abstract. The paper presents a method for determining the effect of teeth correction in an involute worm gear on the contact strength, wear and life of teeth of the worm wheel. As a result of the teeth correction, the profile of the worm wheel teeth undergoes displacement (positive or negative) with respect to the initial profile (uncorrected) and formation of a tooth profile by some other section of the involute. As a result, the contact pressures and teeth wear decrease while the gear life increases. The regularities regarding the effect of correction on contact and tribocontact parameters are established. It has been found that, in contrast to the uncorrected gear, when the correction coefficient x_2 is positive, the maximum contact pressures decrease; when the correction coefficient has a negative value – the pressures increase. The radii of teeth curvature increase with an increase in x_2 and they decrease with a decrease in x_2. The wear of the worm wheel teeth decreases at $x_2 > 0$ and increases at $x_2 < 0$ compared to the case when $x_2 = 0$. The life of the gear increases at $x_2 > 0$ while at $x_2 < 0$ it decreases.

1. Introduction

Archimedes worm gears are widely used in various devices and equipment. On meshing a sliding friction is generated, which leads to the wear of teeth of the worm wheel [1]. For this reason, it is required that the life of such a gear and the wear of its wheel be determined already at the stage of design. The literature reports methods for determining abrasive wear of worm gear teeth [2 - 7]; however, the proposed methods cannot be used for uncorrected and corrected gears. Accordingly, in [2,3] the effect of loading on the contact parameters at EHDL was investigated. The studies [4, 5] report the results regarding determination of teeth wear in an uncorrected gear according to a modified Archard law which takes into account the changes in contact pressures and oil film thickness in the contact zone based on elastohydrodynamic lubrication theory. The behavior of worm gears during loading, including the occurrence of contact pressures, was investigated in [8]. In [9], a model for predicting tooth wear along the contact line is presented. The model takes account of lubricant thickness, which is also based on Archard's law of abrasive wear. A method [10, 11] was developed for determination of contact parameters in uncorrected worm gears; later on, it was generalized and applied to corrected worm gears

2. Research method

2.1. Unit linear wear function

According to [10, 11] the function of liner wear of teeth of a worm gear per one revolution is calculated from the formula

$$h'_{2j}=\frac{\nu_j t'_j\left(fp^{(w)}_{j\max}\right)^{m_2}}{C_2\left(\tau_{s2}\right)^{m_2}}\,, \tag{1}$$

where

$t'_j=2b_j/v_j$ is the time of contact between the meshing components at *j*-th point on the friction path with a length of $2b_j$;

v_j is the velocity of slide at j-th point of meshing at the height of worm coils;

f is the sliding friction coefficient;

C_2, m_2 are the indicators of wear resistance of material of the worm wheel in a selected pair and the conditions of wear as determined in the experimental tests;

$\tau_{s2}\approx 0,35R_{m2}$ is the temporary shear strength of material the worm wheel;

R_{m2} is the temporary tensile strength of material of the worm wheel;

$2b^{(w)}_j=2.256\sqrt{\Theta N'\rho_{zj}/bw}$ is the width of contact area;

$p^{(w)}_{j\max}=0,564\sqrt{N'/w\theta\rho_{zj}b}$ are the maximum contact pressured determined according to the Hertz formula depending on the number of meshing pairs w of teeth of the worm wheel;

w is the number of engaged tooth pairs;

$\theta=\left(1-\mu_1^2\right)/E_1+\left(1-\mu_1^2\right)/E_2$ is the Kirchhoff modulus;

μ, E are the Poisson ratio and Young modulus of materials of the worm wheel, respectively;

b is the worm wheel width;

ρ_{zj} is the radius of curvature of teeth of the worm wheel at j-th point of meshing

$$\rho_{zj}=\frac{\rho_{1j}\rho_{2j}}{\rho_{1j}+\rho_{2j}}\,.$$

2.2. Curvature radius and gear geometry

Curvature radius both of involute worm thread profiles ρ_{1j} and of the worm wheel teeth ρ_{2j}

$$\rho_{1j}=-\frac{r_b tg\alpha_{cj}}{\cos^3\alpha_{pxj}tg\gamma_b\cos^2\left(\alpha_{cj}+\varepsilon_j\right)},\quad \rho_{2j}=\frac{\rho_{1j}r_2\sin\alpha_{pxj}+\rho_{1j}e_{pAj}-e^2_{pAj}}{r_2\sin\alpha_{pxj}+\rho_{1j}-e_{pAj}}; \tag{2}$$

$x_A\langle x\langle x_B, x_A=r_{f_1}+0,2m,\ x_B=r_{a_1}$; $r_{f_1}=0,5\left(d_1-2h_{f_1}\right), h_{f_1}=1,2m$ (where $\gamma\le 15^\circ$), $h_{f_1}=1,2m_n$ (where $\gamma\rangle 15^\circ$); $tg\gamma=mz_1/d_1$, $d_1=qm$; $r_{a_1}=0,5\left(d_1+2h_{a_1}\right)$, $h_{a_1}=m$ (where $\gamma\le 15^\circ$), $h_{a_1}=m_n$ (where $\gamma\rangle 15^\circ$); $r_2=0,5z_2m,\ r_2=0,5d_2,\ z_2=uz_1$; $r_b=0,5d_1\cos\alpha_c$, $tg\alpha_c=tg\alpha_n/\sin\gamma$, $q=2\left(1+\sqrt{z_2}\right)$; $\alpha_n=\alpha=20^\circ$; $\alpha_{cj}=arctg\dfrac{\sqrt{x^2-r_b^2}}{r_b}$;

$$\alpha_{pxj} = arctg\left(-tg\gamma_b \frac{\sqrt{x^2 - r_b^2}}{x}\right); tg\gamma_b = \frac{mz_1}{d_1 \cos\alpha_c}; \varepsilon_j = \frac{180}{\pi} \frac{\sqrt{x^2 - r_b^2}}{r_b};$$

$$e_{pAj} = \frac{r_1 - x}{\sin\alpha_{pxj}},\ r_1 = 0,5d_1,\ b = 2m\sqrt{q+1},$$

where

r_{f1} is the radius of a circle of worm cavity;

d_1 is the reference diameter of the worm;

h_{f_1} is the height of worm thread base;

m is the axial modulus of meshing;

$m_n = m\cos\gamma$ is the normal modulus of meshing;

γ is the angle of elevation of the screw line of worm coils;

z_1 is the number of worm coils;

q is the diametral quotient of the worm gear;

r_{a1} is the radius of a circle of worm coil prongs;

h_{a1} is the height of head of the worm coil;

d_2 is the reference diameter of the worm wheel;

z_2 is the number of teeth in the worm wheel;

u is the gear ratio;

r_b is the radius of a basic circle of worm coils;

α_c is the face pressure angle;

$\alpha_n = \alpha = 20^\circ$ is the angle of meshing;

α_{cj} is the face pressure angle of j-th point;

ε is an angular coordinate in every pitch (degrees);

e_{pA} is the distance of j point from the contact point.

The section of meshing $[x_A, x_B]$ must be divided proportionately into smaller sections: $x_A = j_A = j_1,\ x_2 = j_2,\quad x_3 = j_3,\ ....,\ x_B = j_n = j_B$.

2.3. Slipping velocity

The slipping velocity v_j is determined according to the formula:

$$\nu_j = \frac{\omega_1 x}{\cos\gamma_A}, \tag{3}$$

where $tg\gamma_A = mz_1/2x$; $\omega_1 = \pi n_1/30$ is the angular velocity of the worm; n_1 is the number of revolutions of the worm.

2.4. Wear and life of the worm gear

The wear of the worm wheel teeth within one hour of gear operation is [10]**:**

$$\bar{h}_{2j} = 60 n_2 h'_{2j}, \quad n_2 = n_1 / u, \tag{4}$$

where n_2 is the number of revolutions of the worm wheel per minute; h'_{2j}, $n_2 h'_{2j}$ are the linear wear of the worm wheel teeth within one revolution and one minute of operation, respectively.

The life of the worm gear for the acceptable wear h_{2*} of the worm wheel teeth is calculated according to the formula:

$$t_* = \left(h_{2*} / \bar{h}_{2j}\right) \tag{5}$$

2.5. Meshing force

The meshing force is calculated according to the formula [10]

$$N' = \frac{2T}{d_1 \cos\alpha_{pxj} \sin(\gamma + \rho')}, \tag{6}$$

where $T = 9550 \cdot 10^3 (N / n_1)$ is the torque transmitted by the worm; $\rho' = arctg\left(f / \cos\alpha\right)$ is the friction angle; $N$is the transmitted power.

2.6. Worm wheel teeth correction

As a result of the teeth correction, the profile of the worm wheel teeth undergoes displacement (positive or negative) with respect to the initial profile (uncorrected) and formation of a tooth profile by some other section of the involute. As a result, the contact pressures and teeth wear decrease while the gear life increases. In worm gears, we can only apply correction to the worm wheel teeth. Hence, the applied displacement of the gear-cutting hob is:

$$\xi = x_2 m,$$

where $x_2 \le \pm 1$ is the correction correlation.

The interaxial distance is:

$$a_{wk} = a_w + x_2 m, \tag{7}$$

where $a_w = r_1 + r_2$ is the interaxial distance in the uncorrected gear.

The reference diameter of the worm in the uncorrected gear

$$d_{w1} = d_1 + x_2 m.$$

Therefore, the distance e_{pAj} between the j-th point of contact and the point of contact in [11] will be:

$$e_{pAj} = \frac{r_{w1} - x}{\sin\alpha_{pxj}}, \; r_{w1} = d_{w1} / 2.$$

For the uncorrected worm gear, the force in meshing

$$N' = \frac{2T}{d_{w1} \cos\alpha_{pxj} \sin(\gamma + \rho')}.$$

3. Solution

Other geometrical parameters are determined according to the formulae for the uncorrected worm gear. The computations were performed using the following set of data: N = 3.5 kW, n_1 = 1410 rev/min, m = 6 mm, z_1 = 2, u = 25.5, f = 0.05, q = 8; worm- hardened steel grade 45 (HRC 50) described by $E_1 = 2.1 \cdot 10^5$ MPa, μ_1 = 0.3; worm wheel ring – bronze CuSn6Zn6Pb6 described by $E_2 = 1.1 \cdot 10^5$ MPa, μ_2 = 0.34; $C_2 = 7.6 \cdot 10^6$, m_2 = 0.88; τ_{s2} = 75 MPa; for j = 1; 2; 3; 4 and 5, respectively x = 18; 20; 22; 24 and 26 mm; h_{2*} = 0.5 mm; with double-pair meshing.

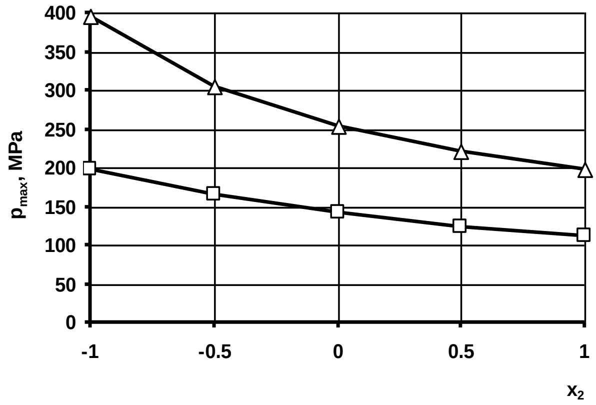


Figure 1. The influence of correction on p_{max}: □ – value p_1 by j = 1; △ – value p_5 by j = 5.

The results of the numerical solution are given in Figures. 1–6. Figure 1 shows the relationships between the maximum contact pressures p_{max} and the displacement coefficient x_2 on entering the mesh (j = 1) and on leaving the mesh (j = 5). It was found that the positive correction of the worm wheel teeth leads to reduced pressures, while the negative correction results in their increase compared to the observations made regarding the uncorrected gear. This results in an increase in the teeth curvature summary radius $\rho_z = \rho$ at their acceptable wear h_{2*} = 0.5 mm when x_2 is positive and it decrease at negative values (Figure 2).

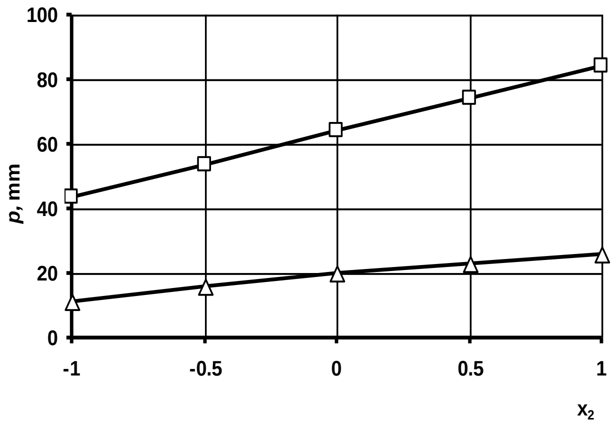


Figure 2. The influence of correction on reduced curvature radii change: □ – value ρ_{zmax} by j = 1; △ – value ρ_{zmin} by j = 5.

The effect of teeth correction of the linear wear $\bar{h}_2$ of the worm wheel teeth during one hour of gear operation is illustrated in Figure 3. Figure 4 shows the minimal life t_{min} of the corrected gear (j = 1; x = 18 mm) at the acceptable wear h_{2*} = 0.3 та 0.5 mm.

As a result of the applied teeth correction, when $x_2 > 0$, ρ_z increases and so the wear $\bar{h}_2$ (Figure 3) decreases and the gear life t_{min} increases (Figure 4); when $x_2 < 0$, ρ_z decreases, which causes consequences which are undesired from a practical point of view. It must be stressed that the linear wear of teeth on entering the mesh (j = 1) and on leaving the mesh (j = 5) is similar (Figure 3), although the pressures p_{max} (see Figure 1) differ to a significant degree.

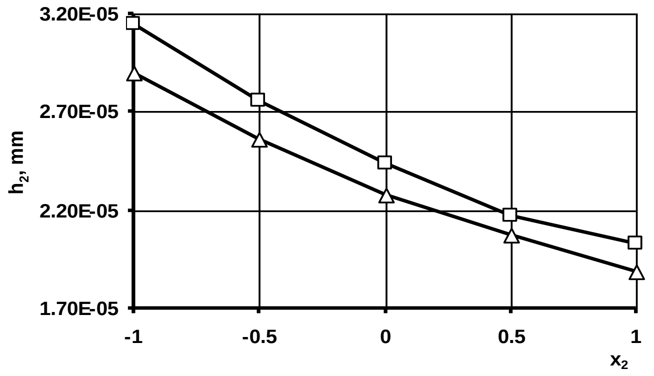


Figure 3. Gear teeth wear versus x_2: □ – value $\overline{h}_{2\max}$ by $j = 1$; △ – value $\overline{h}_{2\min}$ by $j = 5$.

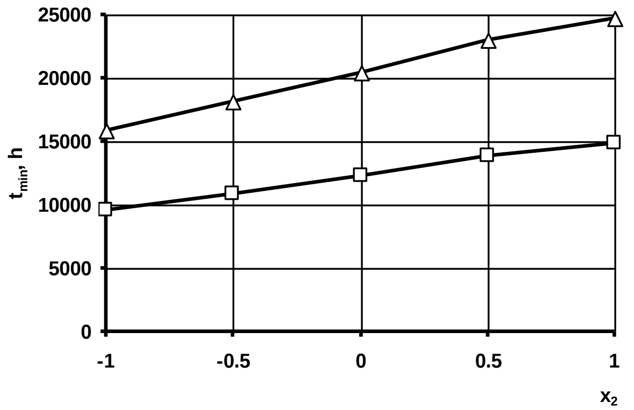


Figure 4. The influence of correction on gear life $t_{\min}$: △ – value $t_{\min}$ by $h_{2*} = 0.3$ mm; □ – value $t_{\min}$ by $h_{2*} = 0.5$ mm.

The applied teeth correction does not affect the sliding velocity v_j in meshing and its value only depends on the position of contact point at the height of the tooth (Figure 5).

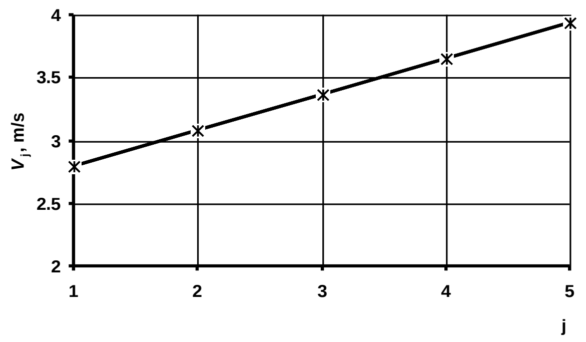
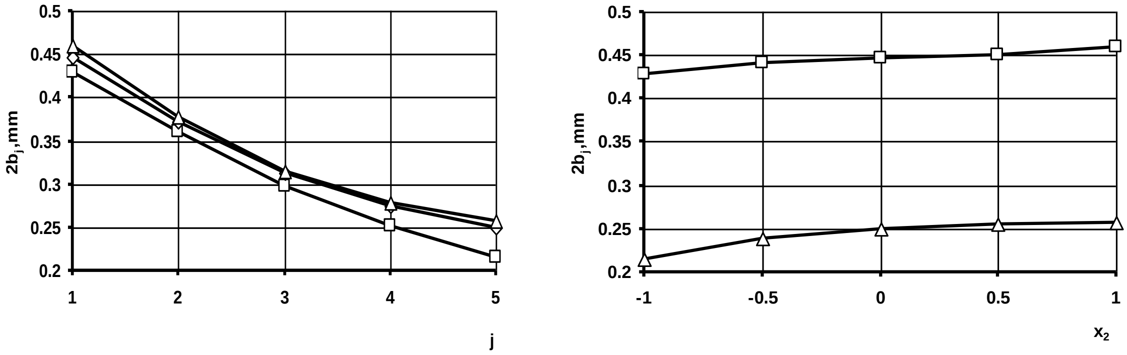


Figure 5. Change in sliding rate over tooth height

We also investigated the contact area width $2b_j$ at meshing points $j = 1, 2, \ldots, 5$ (Figure 6*a*) and the time t'_j of unit contact. It was found that the unit sliding friction path $2b$ decreases significantly from $j = 1$ to $j = 5$, so the life of the gear decreases, too. When the gear teeth enter the mesh, the parameter $2b_1$ remains practically unchanged, but when the teeth leave the mesh the value of $2b_5$ (bottom curve) slightly changes (Figure 6*b*).



a *b*

Figure 6. The influence of correction on contact area per one interaction of teeth and worm wheel revolution: □ – x_2 = 0; ○ – 1; △ – x_2 = -1.

The discovered qualitative and quantitative relationships with respect to the effect of teeth correction on the load capacity, wear, life and sliding velocity of the gear are reported in the Conclusions section below.

Conclusions

1. It has been found that, in contrast to the uncorrected gear, when the correction coefficient x_2 is positive, the maximum contact pressures decrease; when the correction coefficient has a negative value – the pressures increase (Figure 1).

2. The radii of teeth curvature increase with an increase in x_2 and they decrease with a decrease in x_2 (Figure 2).

3. The wear of the worm wheel teeth decreases at $x_2 > 0$ and increases at $x_2 < 0$ compared to the case when $x_2 = 0$ (Figure 3).

4. The life of the gear increases at $x_2 > 0$ while at $x_2 < 0$ it decreases (Figure 4).

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