Parametric sensitivity functions as an instrument for the coordinates determining accuracy improvement in radio monitoring systems

R L Pantyeyev[1,5], P I Bidyuk[2], Yu A Opanasyuk[3] and A O Platunov[4]

[1,2,4] Department of aviation computer-integrated complexes, National Aviation University, 1 Liubomyra Huzara ave., Kyiv, 03058, Ukraine.
[3] Department of mathematical methods of system analysis, Institue for Applied and System Analysis, National Technical University of Ukraine "Kyiv Polytechnic Institute", Peremohy ave, 37, Kyiv, 03056, Ukraine.

[5] E-mail: romanpanteevmail@gmail.com

Abstract. The purpose of this paper is to evaluate the possibility of optimizing the process of calculating the coordinates range difference way in passive radar systems and evaluation of measurement uncertainty through the use of the sensitivity coefficients. It is shown that the sensitivity coefficients are an effective tool for determining the area in which a given probability can be a source of radio emission. In addition, possibility of reducing the number of receiving stations to one and reduce the amount of computation, if one of the coordinates of the radio source remains unchanged.

1. Introduction

Important task by development of radio monitoring systems is the problem of definition of an error of calculation of coordinates of sources of a radio emission. In case of finding of a source of a radio emission (monitoring object) on different removal from the spaced system of a passive radar-location, one of effective ways of definition of absolute error of measurement of coordinates is the method of functions of sensitivity connected with studying of impact of change of input parameters on change of days off. Input parameters in the systems of a passive radar-location are meant as time delays of arrival of a signal of monitoring object at the station of a complex, and by output parameters – monitoring object coordinates calculated by a complex.

By studying dynamic systems the concept of unambiguous compliance between vectors of input and output parameters which can be defined by means of differential equations, equations of state or any other way is often used. However, at practical calculations input parameters can be determined only with some accuracy. Besides, parameters of a system change depending on external conditions and in time that is engineering calculations deal with rated values of parameters and with the corresponding admissions. In this regard, instead of an unambiguous ratio between nominal input and output parameters is more practical to consider a ratio of areas of change of input and output parameters concerning the rated values which give information concerning sensitivity of a system to perturbations [1].

2. Problem statement

Are a part of the considered system four spaced stations C, R, L and Q (fig. 1), and for calculation of coordinates the differential and range-metering method based on measurement of a difference of times of arrival of a signal from monitoring object on side stations in relation to central and creation on them the corresponding hyperboles is used. Location of monitoring object is the cross point of three hyperboloids of rotation [2].

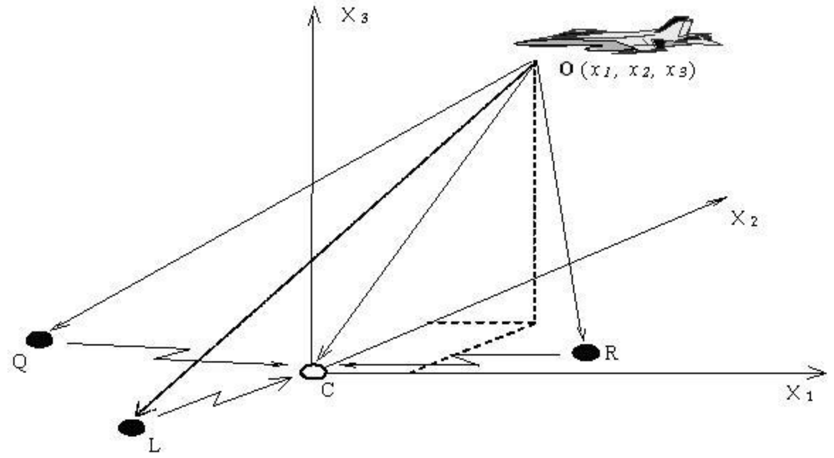

Figure 1. Passive monitoring system of airspace

For determination of coordinates of an object at the moment time needs to solve a system of equations (1), received the name – hyperbolic [3].

$$\begin{aligned} \tau_L &= \frac{1}{c}\cdot\left(\overline{OL}+\overline{LC}-\overline{OC}\right)= f(x_1,x_2,x_3) \\ \tau_R &= \frac{1}{c}\cdot\left(\overline{OR}+\overline{RC}-\overline{OC}\right)= g(x_1,x_2,x_3) \\ \tau_Q &= \frac{1}{c}\cdot\left(\overline{OQ}+\overline{QC}-\overline{OC}\right)= h(x_1,x_2,x_3) \end{aligned} \tag{1}$$

$\tau_{L,R,Q}$ are delays of time of arrival of a signal at the station

c is speed of distribution of a signal.

The system of equations (1) is expressed through coordinates of monitoring object and stations of a complex in a look:

$$\begin{aligned} F_L &= \frac{1}{c}\left(\sqrt{(x_1-x_{1L})^2+(x_2-x_{2L})^2+(x_3-x_{3L})^2}+D_L-\sqrt{x_1^2+x_2^2+x_3^2}\right)-\tau_L=0 \\ F_R &= \frac{1}{c}\left(\sqrt{(x_1-x_{1R})^2+(x_2-x_{2R})^2+(x_3-x_{3R})^2}+D_R-\sqrt{x_1^2+x_2^2+x_3^2}\right)-\tau_R=0 \\ F_Q &= \frac{1}{c}\left(\sqrt{(x_1-x_{1Q})^2+(x_2-x_{2Q})^2+(x_3-x_{3Q})^2}+D_Q-\sqrt{x_1^2+x_2^2+x_3^2}\right)-\tau_Q=0 \\ D_L &= \overline{LC},\ D_R=\overline{RC},\ D_Q=\overline{QC} \end{aligned} \tag{2}$$

x_1, x_2, x_3 are target coordinates

x_{1L}, x_{2L}, x_{3L} are coordinates of station L

x_{1R}, x_{2R}, x_{3R} are coordinates of station R

x_{1Q}, x_{2Q}, x_{3Q} are coordinates of station Q.

Let it is known σ_i^{τ} error (standard deviation) measurements of times of delays $\tau_{L,R,Q}$ receipts of a signal at the station.

Coordinates of monitoring object are calculated by finding of roots of a system of equations (2). On condition of precise measurement of time τ_0, we will receive an exact solution of system (2) $\vec{x}_0$. It is necessary to receive assessment of coordinates of monitoring object at wrong values a vector component $\vec{\tau}=[\tau_1,\tau_2,\tau_3]$, where $\tau_1=\tau_L$, $\tau_2=\tau_Q$, $\tau_3=\tau_R$. For this purpose we will spread out a vector $\vec{x}$ in a row Taylor in the neighborhood of exact coordinates of monitoring object $\vec{x}_0$, being limited to its linear part [3]:

$$\vec{x}(\tau_0+\Delta\tau)=\vec{x}(\tau_0)+\left.\frac{\partial\vec{x}}{\partial\tau}\right|_{\tau=\tau_0}(\Delta\tau), \tag{3}$$

or

$$\vec{x}(\tau_0+\Delta\tau)-\vec{x}(\tau_0)=\left.\frac{\partial\vec{x}}{\partial\tau}\right|_{\tau=\tau_0}(\Delta\tau) \tag{4}$$

Shift value Δx coordinates, caused by a delay period measurement error on value $\Delta\tau$ is defined by sensitivity coefficients by the following ratio:

$$\Delta\vec{x}(\tau_0+\Delta\tau)=\left.\frac{\partial\vec{x}}{\partial\tau}\right|_{\tau=\tau_0}(\Delta\tau) \tag{5}$$

where

$$\left.\frac{\partial\vec{x}}{\partial\tau}\right|_{\tau=\tau_0}=\begin{bmatrix}\frac{\partial x_1}{\partial\tau_1} & \frac{\partial x_1}{\partial\tau_2} & \frac{\partial x_1}{\partial\tau_3}\\ \frac{\partial x_2}{\partial\tau_1} & \frac{\partial x_2}{\partial\tau_2} & \frac{\partial x_2}{\partial\tau_3}\\ \frac{\partial x_3}{\partial\tau_1} & \frac{\partial x_3}{\partial\tau_2} & \frac{\partial x_3}{\partial\tau_3}\end{bmatrix} \tag{6}$$

matrix of the first derivative coordinates of monitoring object from time to time delays $\vec{\tau}=[\tau_1,\tau_2,\tau_3]$ (Jacobean matrix).

Each coordinate x_i vector $\vec{x}$ will receive the corresponding deviation in connection with delay period measurement errors, namely:

$$x_i(t_0+\Delta\tau)=x_i(t_0)+\sum_{j=1}^{3}\frac{\partial x_i}{\partial\tau_j}\Delta\tau_j\,,\quad i=1\ldots3 \tag{7}$$

The ratio (7) contains monitoring object coordinate measurement error proportional to a delay period measurement error $\Delta\tau_j$, proportionality coefficient at the same time is function of sensitivity $\frac{\partial x_i}{\partial\tau_j}$ coordinates x_i to change of delay period. Knowing function of sensitivity and the measured time delay, it is possible to find value of absolute error of the calculated coordinate.

3. Results

Let's consider the option of symmetric location of stations and a source of a radio emission moving along the route (figure 2).

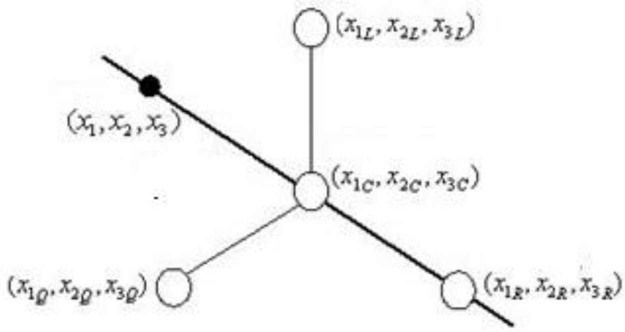

Figure 2. Symmetric location of stations with the route of a source of a radio emission

Let's calculate Jacobi's matrixes at $\delta\tau_i = 15\%$, $i = \overline{1,3}$ at the position of monitoring object over L, Q, R stations and also over station C and behind station C it is symmetric to station R (as shown in fig. 2). Results of calculation are given in table 1.

Table 1. Jacobean matrixes at the provision of monitoring object over L, Q, R stations and also over station C and behind station C it is symmetric to station R, $\delta\tau_i = 15\%$, $i = \overline{1,3}$

	Over station L (measurement τ_1)			Over station Q (measurement τ_2)			Over station R (measurement τ_3)		
	$\frac{\partial x_i}{\partial \tau_1}$	$\frac{\partial x_i}{\partial \tau_2}$	$\frac{\partial x_i}{\partial \tau_3}$	$\frac{\partial x_i}{\partial \tau_1}$	$\frac{\partial x_i}{\partial \tau_2}$	$\frac{\partial x_i}{\partial \tau_3}$	$\frac{\partial x_i}{\partial \tau_1}$	$\frac{\partial x_i}{\partial \tau_2}$	$\frac{\partial x_i}{\partial \tau_3}$
x_1	**5.676*10^3**	5.482*10^5	2.309*10^5	2.107*10^4	**6.424*10^4**	2.088*10^5	3.093*10^5	4.167*10^5	**1.316*10^4**
x_2	**1.703*10^4**	1.76*10^6	2.353*10^5	2.661*10^5	**2.452*10^5**	1.567*10^5	1.2*10^5	2.17*10^5	**1.236*10^4**
x_3	**3.867*10^5**	3.002*10^6	5.212*10^4	3.184*10^5	**1.116*10^5**	1.497*10^4	5.852*10^5	3.89*10^5	**3.07*10^5**

	Over station C			Symmetrically to station R concerning station C		
	$\frac{\partial x_i}{\partial \tau_1}$	$\frac{\partial x_i}{\partial \tau_2}$	$\frac{\partial x_i}{\partial \tau_3}$	$\frac{\partial x_i}{\partial \tau_1}$	$\frac{\partial x_i}{\partial \tau_2}$	$\frac{\partial x_i}{\partial \tau_3}$
x_1	4.218*10^4	1.25*10^3	1.705*10^5	8.495*10^4	6.398*10^4	2.316*10^5
x_2	1.593*10^4	2.201*10^5	1.075*10^5	2.04*10^5	2.828*10^5	1.699*10^5
x_3	5.223*10^5	5.523*10^4	8.116*10^4	7.131*10^5	7.628*10^5	9.04*10^5

It is visible that the smallest values have coefficients of sensitivity when finding monitoring object over station C, that is in the center of a system. In all other cases coefficients of sensitivity have great values, and it will be shown the stronger, than more monitoring object moves away from stations of a complex. Besides, from the table it is visible that when finding monitoring object directly over stations L, Q, R coefficients of sensitivity have the minimum value on that time delay which is defined by this station. It can be explained with the fact that the error entered to calculation of coordinates at change to the corresponding time delay will be small when finding monitoring object over the relevant station in comparison with the entered error in calculation of coordinates by other stations.

The locations of stations of a complex and the route corresponding to a case as shown in fig. 2, functions of sensitivity K_x, K_y, [km/s] depending on coordinates and relative error of measurement of delay period of arrival of a signal, δτi = 15%, are given in fig. 3 and fig. 4.

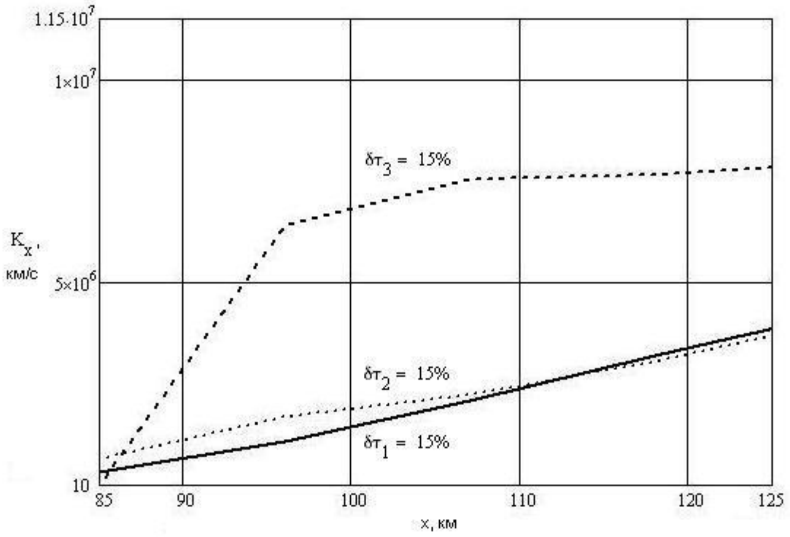

Figure 3. Functions of sensitivity K_x coordinate x at movement along the route at $\delta\tau_{1,2,3}$=15% (functions of sensitivity for $\delta\tau_3$ for combination of diagrams are increased by coefficient 0.05)

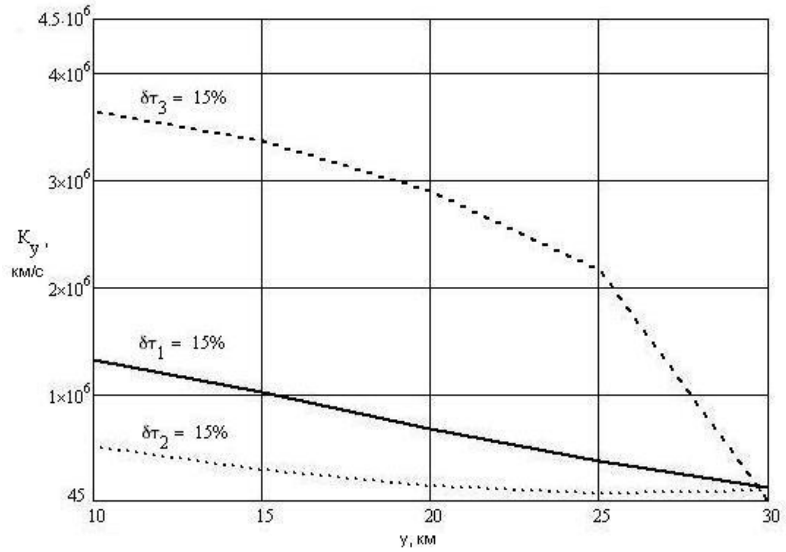

Figure 4. Functions of sensitivity K_y coordinate at at movement along the route at $\delta\tau_{1,2,3}$=15% (functions of sensitivity for $\delta\tau_3$ for combination of diagrams are increased by coefficient 0.2)

4. Discussion

The behavior of these dependences demonstrates that rather little changes of value of time delays lead to significant changes in the calculated coordinates, that is emergence of considerable errors of calculation of coordinates. From here the requirement of the most precise measurement of delays of time of arrival of a signal of stations of a complex follows. Besides, the analysis of these dependences allows to draw a conclusion on existence near stations of a complex of a certain zone, sensitivity coefficients in which have the smallest value and their change in this zone, is insignificant. Outside this zone of function of sensitivity have growth increasing in process of removal from a location of stations.

During creation mathematical and the software of systems of a passive location there is a problem of reduction of volume of computing operations for a solution of information tasks. At a solution of a coordinate and route task reduction of computing operations is reached by the following in the ways [5]:

- use of the modified method of Newton connected with reduction of number of recalculation of a matrix of private derivatives;
- an exception of iterative process of one of the equations of a system (2) if one of coordinates of monitoring object does not change (for example, flight altitude) and also at achievement of given accuracy any of coordinates of provision of monitoring object.

In the second case it is important to define - what equation so and what measurements of the station to exclude from system (2). The analysis of behavior of functions of sensitivity of coordinates of monitoring object concerning a measurement error of delay periods can give the answer to this question τ_i, $i = \overline{1,3}$.

With an invariable height (the third coordinate) for reduction of time of computation process it is necessary to exclude such station for which the value of total sensitivity of coordinates of monitoring object $\sum_{i=1}^{3} \frac{\partial x_i(t)}{\partial \tau_j}$ to an error of definition of delay period τ_j will be the greatest. From table 1 and figures 3, 4 it is visible that during removal of monitoring object from stations of a complex on the route set on figure 2, the greatest values will have sensitivity coefficients at change $\delta\tau_3$, which is measured at station R and, respectively, it can be excluded from process of calculation of coordinates that will have two advantages: will allow to pass from a system from three equations to a system from two equations that will reduce the volume of calculations and also will increase the accuracy of determination of coordinates at the expense of an exception of process of calculations of the station which has the greatest coefficient of sensitivity, and, therefore, will enter the greatest error to calculations.

Conclusions

Determination of coefficients of sensitivity is rather easy way of receiving assessment of a measurement error of coordinates by method by a differential and range-metering method in the systems of a passive radar-location.

Dependences of coefficients of sensitivity on coordinates of air targets and relative error of measurement of delay period of arrival of a signal are received. It is shown that measurement errors of coordinates grow with removal from the location of a system of a passive radar-location.

The algorithm allowing reducing the volume of calculations and time of calculation of coordinates at creation of routes of air targets is offered.

References

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