Hybrid functions entropy doctrine for operational effectiveness: theoretical aspect

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Abstract. The paper considers the theoretical aspects and possibility of the quantitative estimations implementations for the optimal operational effectiveness with the help of the subjective entropy paradigm realized in the Subjective Preferences Entropy Theory through the formulated there Subjective Entropy Maximum Principle. The other kind of optimization, but similar in mathematical apparatus to the mentioned above, is based upon the hybrid-optional effectiveness functions entropy conditional optimization doctrine. Both subjective and objective wings of the general entropy paradigm theory can deal with the specific conflicts of the subjectively preferred operational alternatives preferences functions of the virtual intensions of subjective analysis as well as with the objectively existing options of the real phenomena, which pertains to the ongoing processes and is an intrinsic property of the material world. The described important method with the use of the traditional view entropy allows constructing a hybrid pseudo-entropy relative function that shows the optional side and relative value of the uncertainty or certainty measure of the operational situation. Illustrative examples with the alternatives of a horizontal maximum duration and range flights are demonstrated.

1. Introduction

The presented paper is devoted to the application of the entropy methods to the theoretical aspects of aircraft operational effectiveness. It is proposed to consider the entropy based optimization approaches to the spheres of the flight operation, maintenance techniques, decision making etc. paying attention to the situations of multi-alternativeness and multi-“optionallity” for them. Entropy methods can be significantly helpful as it seems to the application fields with the extensively high levels of uncertainties.

Last decades, the elaborations have been dealing with the certain variational principles implemented into the spheres of reliability, flight safety, aircraft and its parts, systems, and elements maintenance and repair. The entropy concept has been implemented to all those areas.

2. Survey of existing problems

First of all it is necessary to make a few remarks to the connections between the entropy concepts in the framework of the general entropy paradigm.

The traditional Shannon’s view entropy for probabilities of events and probabilities of random values magnitudes was used in theoretical physics [1-3]. There, that method has got the name of the Jaynes’ Entropy Maximum Principle; and after it was implemented in a variety of problems. Later on, the similar mathematical approach was applied to psychological issues of the so-called active systems in subjective analysis [4] where that approach has been renamed as the Subjective Entropy Maximum Principle. In the interpretations of the subjective preferences functions uncertainty measure of individuals [4] that Entropy Principle considerably widened the application abilities of the theoretical studies. It became obvious that subjective uncertainties can be assessed; and preferences functions of alternatives were being obtained in an explicit view trough the entropy conditional optimization. Nowadays, the willing to combine the intrinsic objectively existing features, likewise possibly prospectively applicable to aviation industry [5-15], with the postulated in subjective analysis optimal distributions of the preferences functions has instigated and will definitely continue to instigate the scientific research initiated in references [16-20]. Such combination of the objectively existing options with the subjectively preferred alternatives is being developed presently under the conventional name of the hybrid-optional effectiveness functions entropy conditional optimization doctrine. Since the theory formation has not been finished yet, it needs more illustrations of the theoretical aspects implementations. And that is the presented paper's major objectives to demonstrate the examples applications.

3. Ideas of the subjective entropy maximum principle

Based upon the Jaynes' Entropy Maximum Principle [1-3], the subjective analysis [4] postulates the Subjective Entropy Maximum Principle expressed with the general view functional:

$$\Phi_{\pi} = \alpha H_{\pi} + \beta \varepsilon + \gamma \mathscr{N} , \tag{1}$$

where α, β, γ are corresponding structure parameters that can be considered at different problems settings as the Lagrange coefficients or weight coefficients. Here they are interpreted as internal cyber object control parameters which reflect certain properties of the cyber object "attitude" to the achievable alternatives. H_{π} is entropy of the alternatives preferences π; ε is function of the effectiveness that together with the alternatives preferences entropy H_{π} determines conditions of the attainable alternative preferences π distribution optimality; N is normalizing condition, [4].

The problem is to find the available alternatives preferences π optimal distribution on the conditions formulated as the objective functional (2). The first member in the objective functional (3) is the traditional view Shannon's entropy (corresponding α, β, γ structure parameters that can be reduced over α; thus β and γ coefficients get the renewed values, however, for the perceptional ease with the previous designations) transformed for the preferences likewise in the reference of [4]:

$$H_{\pi} = -\sum_{i=1}^{N} \pi_i \ln \pi_i , \tag{2}$$

where i is number-index of the corresponding attainable but sometimes conflicting alternative; N is the total number of the conflicting alternatives; π_i is preferences functions of the i-th conflicting alternatives under consideration.

Optimization procedure for (4) with (2) results in the so-called canonical explicit expressions for the preferences functions of [4]:

$$\pi_i(t) = \frac{\exp\left[-\beta_i F_i\right]}{\sum\limits_{j=1}^{N} \exp\left[-\beta_j F_j\right]} , \tag{3}$$

where $\pi_i(t)$ is the element of generalization when the preference functions of π_i can be considered as the functions of time t; β_i and β_j are one more element of generalization, these are the corresponding values of the structure parameters that relate to those ones of the objective functional of

(5) [4]; F_i and F_j are effectiveness functions of the i-th and j-th achievable but conflicting alternatives correspondingly.

In case of (3) the second member in the objective functional (1) should be as

$$\beta\varepsilon = -\sum_{i=1}^{N} \beta_i \pi_i(t) F_i(t). \tag{4}$$

There can be another kind of generalization for [4], and the procedures of (1-4), when the integral form of the objective functional is considered since there are functions of time t: $\pi_i(t)$ and $F_i(t)$. Therefore, the aspects of the dynamical evolution are taken into account:

$$\Phi_\pi = \int_{t_1}^{t_2} \left(-\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^{N} \pi_i(t) F_i + \gamma \left[\sum_{i=1}^{N} \pi_i(t) - 1 \right] \right) dt. \tag{5}$$

Here in (5), β and γ are corresponding values of the structure parameters that relate to those ones of the objective functional (1), although in the presented view notation they are already reduced by α; F_i is supposedly the effectiveness functions of the i-th achievable alternatives are the functions of time t.

Thus, the second member of (5), analogously to the integral of the entropy taken in the view of (2), relates with some mean magnitude of the effectiveness function ε (see equation (1)) value for the period of integration $[t_1, \ldots, t_2]$.

4. Example of the flight operation alternatives

There might be a specific two-alternative situation with the problem of the choice for a horizontal flight for the maximum range versus a horizontal flight for the maximum duration, either totally for the whole flight or for some flight segments depending upon the purpose of the flight task.

Accordingly, there are two alternative flight speeds:

$$v_T(m) = \sqrt[4]{\frac{4}{3} \frac{bm^2 g^2}{C_{x_0} \rho^2 S^2}} \quad \text{and} \quad v_L(m) = \sqrt[4]{4 \frac{bm^2 g^2}{C_{x_0} \rho^2 S^2}}, \tag{6}$$

where $v_T(m)$ is aircraft optimal speed of a horizontal flight for maximum duration as a function of the aircraft changeable mass m; b is aircraft aerodynamic coefficient; g is the gravity force acceleration; C_{x_0} is aircraft aerodynamic drag coefficient when the lifting force equals "zero" value; ρ is density of the air; S is characterization area of the aircraft; $v_L(m)$ is aircraft optimal speed of a horizontal flight for maximum range as a function of the aircraft changeable mass m.

From the simplest view differential equations of the aircraft motion, one can get the differentials of the horizontal flight duration and range. In the specified idealized conditions for the horizontal flight duration, for instance, the differential is

$$dt = -\frac{2\eta Q \rho v S}{C_{x_0} (\rho v^2 S)^2 + b(2mg)^2} dm, \tag{7}$$

where η is efficiency of the propulsive complex, a constant for the rough problem formulation; Q is low calorific value of the fuel by its working mass; v is aircraft speed.

The corresponding to the duration differential (7) integral for the determination of the maximum duration of the horizontal flight will be

$$T=\int\limits_{M_0}^{M_E}-\frac{2\eta Q\rho vS}{C_{x_0}\left(\rho v^2 S\right)^2+b(2mg)^2}dm, \tag{8}$$

where M_0 is mass of the flying apparatus at the initial moment in time (at the point of the airplane coming up to the straight line horizontal trajectory) M_E is mass of the flying apparatus at the end of the active segment of the horizontal flight, that is at the end of the engine run.

Determination of $v=v(m)$ from the functional (8), from the mathematical point of view, is the simplest problem of the calculus of variations, which has the optimal speed for the flight with the maximum duration as the problem corresponding solution, the first equation of (6). Analogically to (7, 8) approach, the optimal solution of the maximum range horizontal flight, the second equation of (6) is being found. For the problem of the maximum flight duration control (7, 8), for the simplest case on condition of the two specified achievable alternatives, it will be, similar to (5):

$$\Phi_\pi=\int\limits_{M_0}^{M_E}\left\{H_\pi-\beta\left[\pi_1\frac{2\eta Q\rho v_1 S}{C_{x_0}\left(\rho v_1^2 S\right)^2+b(2mg)^2}+\pi_2\frac{2\eta Q\rho v_2 S}{C_{x_0}\left(\rho v_2^2 S\right)^2+b(2mg)^2}\right]+\gamma\left[\sum_{i=1}^{2}\pi_i-1\right]\right\}dm. \tag{9}$$

where H_π is active element's (individual's) subjective preferences entropy, likewise (2):

$$H_\pi=-\sum_{i=1}^{2}\pi_i\ln\pi_i\ ; \tag{10}$$

v_i is alternative speed functions.

The key point to the flight operation problem described with the procedures of the expressions of (6-10) is that at the given alternative speed functions v_i, the optimal distribution of the alternatives preferences functions π_i demonstrates the preferences higher values for the closer to the extremal (6) alternative speed functions values v_i even without determination of that optimal speed solution. One more feature here is that, if one of the alternative speed functions v_i is not given, then solving the system of the Euler-Lagrange equations (the necessary conditions for an extremal to exist) leads to the optimal solution in the view of that not given function, the first one of the equations of (6); i.e. the free function of v (unknown optimal speed of the horizontal flight with regards to the time (duration) of the flight), like the preferences functions of π_i, and, together with them are the optimal solutions (extremals) obtained from the system of

$$\frac{\partial R^*}{\partial\pi_i}-\frac{d}{dm}\left(\frac{\partial R^*}{\partial\pi_i'}\right)=0, \qquad \frac{\partial R^*}{\partial v}-\frac{d}{dm}\left(\frac{\partial R^*}{\partial v'}\right)=0, \tag{11}$$

where R^* is under-integral function (integrand) of the corresponding objective functional taken in the view of the integral (9); π_i' is derivatives of the preferences functions with respect to the aircraft changeable (varying) mass, that is

$$\pi_i'=\frac{d\pi_i}{dm}\ ; \tag{12}$$

v_{opt}' is derivative of the speed with respect to the mass, i.e.

$$v' = \frac{dv}{dm}\,. \tag{13}$$

$$\frac{\partial R^*}{\partial \pi_i'} \equiv 0 \quad \text{and} \quad \frac{\partial R^*}{\partial v_{opt}'} \equiv 0 \tag{14}$$

for this special case considered. Hence, the system (11) is simply

$$\frac{\partial R^*}{\partial \pi_i} = 0\,, \qquad \frac{\partial R^*}{\partial v} = 0\,. \tag{15}$$

5. Alternatives of maintenance techniques

When considering alternatives of aircraft maintenance techniques, there are possibilities of the alternatives assessment, which is the analogy to (1-15) with the estimation of the subjectively preferred effectiveness functions. But, for the options, also, there is a way of the hybrid-optional effectiveness functions entropy conditional optimization doctrine dealing with the objectively existing intrinsic phenomena of the real processes going on in the systems.

The maximum probability of the system state can be determined based upon the entropy conditional optimization for the system's available options (these are the objective stuff with the probabilities of the states [19, 20] on the contrary to (rather than) the subjectively preferred alternatives of the subjective analysis theory [4]). One more problem is the determination of the uncertainty or certainty measure with the relative indication of a "good" and "bad" or "right" versus "wrong" alternative inclinations. Here it is proposed to implement a hybrid pseudo-entropy relative function [16]:

$$\overline{H}_{\max - \frac{\Delta\pi}{|\Delta\pi|}} = \frac{H_{\max} - H_{\pi}}{H_{\max}} \frac{\Delta\pi}{|\Delta\pi|} = \frac{H_{\max} + \sum\limits_{i=1}^{N} \pi(\sigma_i) \ln \pi(\sigma_i)}{H_{\max}} \frac{\left[\sum\limits_{j=1}^{M} \pi\left(\sigma_j^+\right) - \sum\limits_{k=1}^{L} \pi\left(\sigma_k^-\right)\right]}{\left|\sum\limits_{j=1}^{M} \pi\left(\sigma_j^+\right) - \sum\limits_{k=1}^{L} \pi\left(\sigma_k^-\right)\right|}\,, \tag{16}$$

where $H_{\max}$ is entropy maximum value [4]:

$$H_{\max} = \ln N\,; \tag{17}$$

$\Delta\pi$ is factor or index of the alternatives preferences prevalence or domination, introduced in [16]:

$$\Delta\pi = \sum_{j=1}^{M} \pi\left(\sigma_j^+\right) - \sum_{k=1}^{L} \pi\left(\sigma_k^-\right), \tag{18}$$

where σ_j^+ is positive and σ_k^- is negative conflicting alternatives respectably; M is the number of the positive alternatives; L is the number of the negative conflicting alternatives in respect, [16]:

$$M + L = N\,. \tag{19}$$

Conclusions

The background of the entire presented theory (1-19) is the **"Subjective Entropy Maximum Principle"** [4] developed in pursuit of the well-known from the theoretical physics kinetic theory Jaynes' principle [1-3]. It has been successfully adopted as a tool for the aircraft operational problems solutions based upon plausible theoretical explanations and expedient substantiated reasons. As a result, engineering branch of science gets a possibility of the quantitative estimation of the optional effectiveness functions optimal distributions as well as numerical solutions to many problems dealing with the aircraft operation.

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